AN ADAPTIVE FUZZY SLIDING MODE CONTROLLER APPLIED TO A CHAOTIC PENDULUM

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Introduction

Chaotic response is related to a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaos has sensitive dependence to initial condition, which implies that the system evolution may be altered by small perturbations. Chaos control is based on the richness of chaotic behavior and may be understood as the use of tiny perturbations for the stabilization of an UPO embedded in a chaotic attractor. It makes this kind of behavior to be desirable in a variety of applications, since one of these UPO can provide better performance than others in a particular situation. Due to these characteristics, chaos and many regulatory mechanisms control the dynamics of living systems. Inspired by nature, it is possible to imagine situations where chaos control is employed to stabilize desirable behaviors of mechanical systems. Under this condition, these systems would present a great flexibility when controlled, being able to quickly change from one kind of response to another. Literature presents some contributions related to the analysis of chaos control in mechanical systems. Andrievskii and Fradkov (2004) and Savi et al. (2006) present an overview of applications of chaos control in various scientific fields.

There are different techniques employed to perform chaos control (Savi et al., 2006), however, the inspirational idea of these methods is the well-known OGY method (Ott et al., 1990), which is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit. This contribution proposes a robust controller that can be applied to stabilize UPOs of chaotic attractors. The adopted approach is based on the sliding mode control strategy and enhanced by a stable adaptive fuzzy inference system to cope with modeling inaccuracies and external disturbances that can arise. The boundedness of all closed-loop signals and the convergence properties of the tracking error are analytically proven using Lyapunov's direct method and Barbalat's lemma. The general procedure is applied to a nonlinear pendulum that presents chaotic response (De Paula et al., 2006). Numerical simulations are carried out showing the stabilization of some UPOs of the chaotic attractor showing an effective response, demonstrating the controller performance.

Chaotic pendulum

The nonlinear pendulum consists of an aluminum disc with a lumped mass that is connected to a rotary motion sensor. This assembly is driven by a string-spring device that is attached to an electric motor and also provides torsional stiffness to the system. A magnetic device provides an adjustable dissipation of energy. An actuator provides the necessary perturbations to stabilize this system by properly changing the string length. It is assumed that system dissipation may be expressed by a combination of a linear viscous dissipation together with dry friction. Therefore, denoting the angular position as ϕ, the following equation is obtained.

$$\ddot{\phi} + \frac{\zeta}{I}\dot{\phi} + \frac{kd^2}{2I}\phi + \frac{\mu\,\mathrm{sgn}(\dot{\phi})}{I} + \frac{mgD\sin(\phi)}{2I} = \frac{kd}{2I}\left(\sqrt{a^2+b^2-2ab\cos(\omega t)} - (a-b) - \Delta l\right) \tag{1}$$

where ω, a, b, D, d, m, ζ, μ, I, k are system parameters; g is the gravity acceleration and $\mathrm{sgn}(x)$ is the sign of the variable x. De Paula et al. (2006) show that this mathematical model is in close agreement with experimental data and, therefore, it will be used for the control purposes.

Adaptive fuzzy sliding mode control

In order to write Eq. (1) in a more convenient form, it is rewritten as follows:

$$\ddot{\phi} = f(\dot{\phi}, \phi, t) + hu + p \tag{2}$$

where $h = kd/2I$, $u = -\Delta l$, f can be obtained from Eq. (1) and Eq. (2), and the added term p represents both unmodeled dynamics and external disturbances.

Now, let $S(t)$ be a sliding surface defined in the state space by the equation $s(\dot{e}, e) = 0$, with the function $s : \mathbb{R}^2 \to \mathbb{R}$ satisfying $s(\dot{e}, e) = \dot{e} + \lambda e$, where $e = \phi - \phi_d$ is the tracking error, $\dot{e}$ is the first time derivative of e, ϕ_d is the desired trajectory and λ is a strictly positive constant. The controlling of the system dynamics (2) is done by assuming a sliding mode based approach, defining a control law composed by an equivalent control $\hat{u} = \hat{h}^{-1}(-\hat{f} - \hat{p} + \ddot{\phi}_d - \lambda\dot{e})$ and a discontinuous term $-K\,\mathrm{sgn}(s)$:

$$u = \hat{h}^{-1}(-\hat{f} - \hat{p} + \ddot{\phi}_d - \lambda\dot{e}) - K\,\mathrm{sgn}(s) \tag{3}$$

where $\hat{h}$, $\hat{f}$, and $\hat{p}$ are estimates of h, f and p, respectively, and K is a positive control gain.

Regarding the development of the control law, the following assumptions should be made:

Assumption 1 *The function f is unknown but bounded, i.e. $|\hat{f} - f| \leq \mathcal{F}$.*

Assumption 2 *The input gain h is unknown but positive and bounded, i.e. $0 < h_{\min} \leq h \leq b_{\max}$.*

Assumption 3 *The perturbation $p(t)$ is time-varying and unknown but bounded, i.e. $|p(t)| \leq \mathcal{P}$.*

Based on Assumption 2 and considering that the estimate $\hat{h}$ could be chosen according to the geometric mean $\hat{h} = \sqrt{h_{\max}h_{\min}}$, the bounds of h may be expressed as $\mathcal{H}^{-1} \leq \hat{h}/h \leq \mathcal{H}$, where $\mathcal{H} = \sqrt{h_{\max}/h_{\min}}$.

Under this condition, the gain K should be chosen according to

$$K \geq \mathcal{H}\hat{h}^{-1}(\eta + |\hat{p}| + \mathcal{P} + \mathcal{F}) + (\mathcal{H} - 1)|\hat{u}| \tag{4}$$

where η is a strictly positive constant related to the reaching time.

At this point, it should be highlighted that the control law (3), together with (4), is sufficient to impose the sliding condition

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{5}$$

and, consequently, the finite time convergence to the sliding surface S.

In order to obtain a good approximation to the disturbance $p(t)$, the estimate $\hat{p}$ will be computed directly by an adaptive fuzzy algorithm. The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in an appropriate linguistic manner. Considering that each rule defines a numerical value as output $\hat{P}_r$, the final output $\hat{p}$ can be computed by the dot product:

$$\hat{p}(s) = \hat{\mathbf{P}}^{\mathrm{T}}\mathbf{\Psi}(s) \tag{6}$$

where $\hat{\mathbf{P}} = [\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_N]^{\mathrm{T}}$ is the vector containing the attributed values $\hat{P}_r$ to each rule r, $\mathbf{\Psi}(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_N(s)]^{\mathrm{T}}$ is a vector with components $\psi_r(s) = w_r/\sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule. In order to obtain the better estimation $\hat{p}(s)$ to the disturbance p, the vector of adjustable parameters can be automatically updated by $\dot{\hat{\mathbf{P}}} = \varphi s\mathbf{\Psi}(s)$, where φ is a strictly positive constant related to the adaptation rate.

In order to evaluate the stability of the closed-loop system, let a positive-definite function V be defined as

$$V(t) = \frac{1}{2}s^2 + \frac{1}{2\varphi}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta} \tag{7}$$

where $\boldsymbol{\delta} = \hat{\mathbf{P}} - \hat{\mathbf{P}}^*$ and $\hat{\mathbf{P}}^*$ is the optimal parameter vector, associated with the optimal estimate $\hat{p}^*(s)$. Thus, by considering the time derivative of V and defining a minimum approximation error as $\varepsilon = \hat{p}^*(s) - p$, and recalling the definitions of s, u and $\dot{\hat{P}}$, it is possible to verify that $\dot{V}$ becomes

$$\dot{V}(t) = -\Big[(\hat{f} - f) + \varepsilon + \hat{h}^{-1}\hat{u} - h\hat{u} - hK\mathrm{sgn}(s)\Big]s \leq -\eta|s| \tag{8}$$

Here assumptions 1–3 are evoked and K is defined according to (4). This implies $V(t) \leq V(0)$ and that s and $\boldsymbol{\delta}$ are bounded. Integrating both sides of (8) and evoking Barbalat's lemma it is established that $s \to 0$ as $t \to \infty$, which ensures the convergence of the states to the sliding surface S and to the desired trajectory. At this point, it should be notice that discontinuous terms can produce undesirable high frequency oscillations of the controlled variable. Therefore, it is convenient to use saturation functions that smoothes system discontinuities (Slotine and Li, 1991).

Numerical simulations

In order to analyze the controller perfomance, numerical simulatations are carried out considering the fourth order Runge-Kutta method. The model parameters are chosen according to De Paula et al. (2006) and control parameters are $\lambda = 10.0$, $\eta = 0.5$, $\gamma = 1.0$ and $\mathcal{H} = 1$. Basically, two different situations are treated. In the first case, Figure 1, a generic orbit $[\dot{\phi}_d, \phi_d] = [4.6\pi\cos(2\pi t), 2.3\sin(2\pi t)]$ are considered, while in the second case, Figure 2, a period-1 UPO are chosen. Although both orbits are similar, it should be highlighted that the control action u the controller requires less effort to stabilize an UPO.

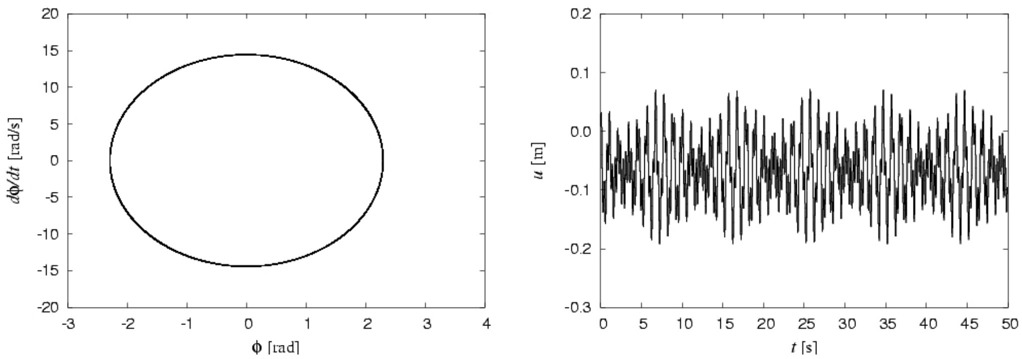

Figure 1: Tracking of $[\dot{\phi}_d, \phi_d] = [4.6\pi\cos(2\pi t), 2.3\sin(2\pi t)]$.

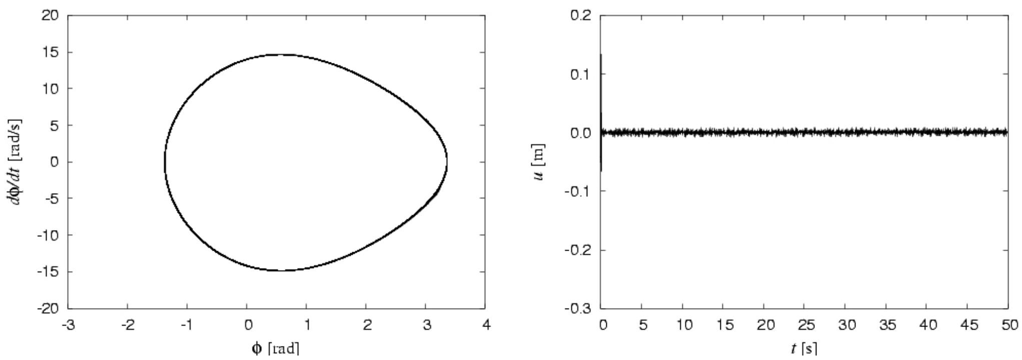

Figure 2: Tracking of a Period-1 UPO.

Concluding remarks

The present contribution considers the stabilization of orbits employing an adaptive fuzzy sliding mode controller. The stability and convergence properties of the closed-loop systems is proven using Lyapunov stability theory and Barbalat's lemma. As an application of the general formulation, numerical simulations of a nonlinear pendulum with chaotic response is of concern. The control system performance is investigated showing for the tracking of a generic orbit as well as for UPO stabilization. It is shown that the controller needs less effort to stabilize an UPO.

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