Performances and Stability Analysis of Networked Control Systems

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Abstract: The problem of performance and stability analysis of networked control systems with random network delay and data dropout is considered in this paper. A new control scheme is proposed to overcome the effects of network transmission delay and data dropout, which is termed networked predictive control. Two different ways to choose control input u_t are discussed in the paper and the results of performances and stability are presented. Both real-time simulations and practical experiments show the effectiveness of the control scheme.

Key Words: Networked control system, predictive control, random network delay, stability.

1 INTRODUCTION

In recent years, more and more network technology has been applied to control systems [1] ,much attention has been paid to the study of control design and stability analysis of networked control system (NCS) [2], [3], [4], The NCS is defined as a feedback control system where control loops are closed through a real-time network [5],[6], [7], [8], [9], [10]. As the structure of networked control systems is different from that of tradition control systems, there exist various specific problems in networked control systems, for example, network delay, loss of data packets, network security and safety [11].

Although much research work has been done in networked control systems, most work has ignored a very important feature of networked control systems. This feature is that the communication networks can transmit a packet of data at the same time, which is not done in traditional control systems [12], [13]. This paper makes full use of this network feature and proposes a new networked control scheme - networked predictive control, which can overcome the effects caused by network delay. Furthermore, two ways to choose control input u_t are discussed in the paper and the performances have been compared and analyzed.

2 NETWORKED PREDICTIVE CONTROL FOR SYSTEMS WITH NETWORK DELAY

This paper considers the case where the system controller is far away from the plant but the sensor is near to the plant. So, the network delay in the feedback channel is not considered. A networked predictive control scheme for NCS with random network delay in the forward channel is proposed. The main part of the scheme is the networked predictive controller, which compensates for the network delay in the feedback channel and achieves the desired control performance.

Consider a MIMO discrete system described in the following state space form

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ y_t &= Cx_t \end{aligned} \tag{1}$$

where $x_t \in R^n, u_t \in R^m$, and $y_t \in R^l$ are the state, input, and output vectors of the system,respectively, $A \in R^{n*n}, B \in R^{n*m}$,and $C \in R^{l*n}$ the system matrices. For the simplicity of stability analysis, it is assumed that the reference input of the system is zero. Also, the following assumptions are made.

Assumption 1: The pair, (A,B), is completely controllable, and the pair, (A,C) is completely observable.

Assumption 2: The number of consecutive data dropouts is less than N_1, where N_1 is a positive integer.

Assumption 3: The upper bound of the network delay is not greater than N, where N is a positive integer.

The state observer is designed as

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t-1} + Bu_t + L(y_t - C\hat{x}_{t|t-1}) \tag{2}$$

where $\hat{x}_{t+1|t} \in R^n$ and $u_t \in R^m$ are the one-step ahead state prediction and the input of the observer at time t, respectively. The matrix $L \in R^{n*l}$ can be designed using

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observer design approaches.
The estimator of the state

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + M(y_t - C\hat{x}_{t|t-1}) \qquad (3)$$

where $\hat{x}_{t|t-1} \in R^n$ is the state observer.

Following the state observer described by (2), based on the output data up to $t+N-1$, the state predictions from time $t+N$ to t are constructed as

$$\begin{aligned} \hat{x}_{t+1|t} &= A\hat{x}_{t|t-1} + Bu_t + L(y_t - C\hat{x}_{t|t-1}) \\ \hat{x}_{t+2|t} &= A\hat{x}_{t+1|t} + Bu_{t+1|t} \\ &\vdots \\ \hat{x}_{t+N|t} &= A\hat{x}_{t+N-1|t} + Bu_{t+N-1|t} \end{aligned} \qquad (4)$$

In particular, the augmented system without time-delay, i.e.,ut=ut|t, can be described as follows:

$$\begin{aligned} x_{t+1} &= (A+BKMC)x_t + (BK-BKMC)\hat{x}_{t|t-1} \\ \hat{x}_{t+1|t} &= (A+BK-LC-BKMC)\hat{x}_{t|t-1} + (BKMC+LC)x_t \end{aligned} \qquad (5)$$

While designing of the Networked Predictive controller , control input u_t is absolutely necessarily. There are three different ways to get u_t ,idiographic cases are analyzed as follows:

While designing of the Networked Predictive controller , control input u_t is absolutely necessarily. There are two different ways to get u_t ,idiographic cases are analyzed as follows:

Case 1: Use estimated value $u_{t|t}$ as the control input, which is shown in Figure 1.

$$u_t = u_{t|t} = K\hat{x}_{t|t} \qquad (6)$$

where $K \in R^{m*n}$ is the state feedback control matrix to be determined using modern control theory. Then, the control predictions are generated by

$$u_{t+k|t} = K\hat{x}_{t+k|t}, \text{ for k = 0,1,2,}\cdots\text{,N} \qquad (7)$$

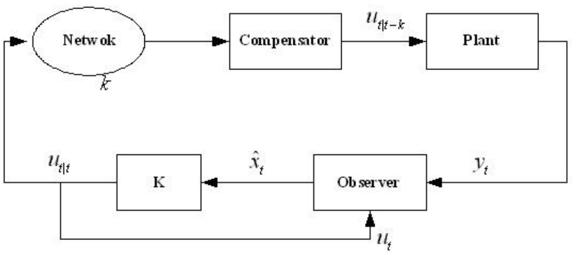

Figure1: The diagram of case 1

Thus, it follows from equation (4) that

$$\hat{x}_{t+k|t} = (A+BK)^{k-1}\hat{x}_{t+1|t} \qquad (8)$$

Based on equations (2), (3) and (6), it can be shown that

$$\begin{aligned} \hat{x}_{t+k|t} &= (A+BK)^{k-1}(BKMC+LC)x_t + \\ &(A+BK)^{k-1}(A+BK-LC-BKMC)\hat{x}_{t|t-1} \end{aligned} \qquad (9)$$

and

$$\begin{aligned} u_{t+k|t} &= K\hat{x}_{t+k|t} \\ &= K(A+BK)^{k-1}(BKMC+LC)x_t + \\ &K(A+BK)^{k-1}(A+BK-LC-BKMC)\hat{x}_{t|t-1} \end{aligned} \qquad (10)$$

Case 2: The controller is designed using $u_{t|t-k}$ as u_t which is sent to the plant directly, as shown in Figure 2:

$$u_t = u_{t|t-k} = K\hat{x}_{t|t-k} \qquad (11)$$

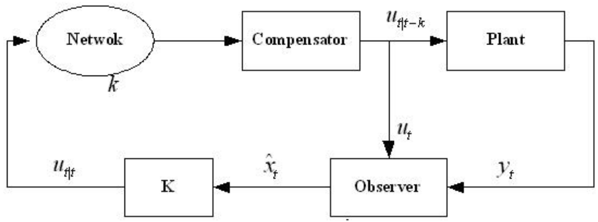

Figure2: The diagram of case 2

where the state feedback matrix $K \in R^{m*n}$, K is the network delay in forward channel.

Using predicted method, the output of the networked predictive control at time t is determined by

$$\begin{aligned} u_t = u_{t|t-k} &= K(A+BK)^{k_t-1}(A-LC)\hat{x}_{t-k|t-k+1} \\ &+ K(A+BK)^{k-1}LCx_{t-k} + K(A+BK)^{k-1}Bu_{t-k} \end{aligned} \qquad (12)$$

3 STABILITY ANALYSIS OF CLOSED NETWORKED PREDICTIVE CONTROL SYSTEMS

Case1:

Theorem 1: For the networked predictive control systems with random network delay in the forward channel, the closed-loop system (1) is stable if there exists a positive definite matrix $P \in R^{(2N+2)n \times (2N+2)n}$ such that

$$\overline{A}^T(k_t)P\overline{A}(k_t) - P < 0 \qquad (13)$$

for $k_t = 0,1,2,\cdots,N$, where

$$\overline{A}(k_t) = \begin{bmatrix} A & 0 & \cdots & M_1(k_t) & 0 & \cdots & 0 & 0 & 0 & \cdots & M_2(k_t) & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & I & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & I & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ M_3(k_t) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & M_4(k_t) & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & I & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & I & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & I & 0 \end{bmatrix}$$

with $k_t \in \{1,2,\cdots,N\}$, $A(k_t) \in R^{2(N+1)n \times 2(N+1)n}$

$M_1(k_t) = BK(A+BK)^{k_t-1}(BKMC+LC)$,

$$M_2(k_t) = BK\left(A + BK\right)^{k_t - 1}\left(A + BK - LC - BKMC\right),$$
$$M_3(k_t) = BKMC + LC\ ,$$
$$M_4(k_t) = A + BK - LC - BKMC\ . \qquad (14)$$

In the case of the closed-loop system without time-delay, we can get $\overline{A}(0)$ as follows:

$$\overline{A}(0) = \begin{bmatrix} M_1(0) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & M_2(0) & \cdots & 0 & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & I & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & I & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ M_3(0) & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & M_4(0) & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & I & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & I & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & I & 0 \end{bmatrix}$$

Where

$$A(0) \in R^{2(N+1)n \times 2(N+1)n},\ M_1(0) = A + BKMC\ ,$$
$$M_2(k_t) = BK - BKMC\ ,\ M_3(k_t) = BKMC + LC\ ,$$
$$M_4(k_t) = A + BK - LC - BKMC$$

Theorem 1 is the same as Theorem 5 in [12].

Case2:

Then, the main theorem 2 is similar to [13] which can be stated as follows:

Theorem 2: For the networked predictive control systems (1) with random network delay in the forward channel, the closed-loop system is stable if there exists a positive definite matrix $P \in R^{(2nN+mN+2n)\times(2nN+mN+2n)}$ such that

$$\Lambda^T(k_t)P\Lambda(k_t) - P < 0 \qquad (15)$$

for $k_t = 0, 1, 2, \cdots, N$ and $\Lambda^T(k_t) = \Lambda^T(k_t, 0)$ which come from $\Lambda^T(k_t, f_t)$ in (20) with $f_t = 0$. The proof is similar to that in [13].

4 REALTIME SIMULATION AND PRACTICAL EXPERIMENT

Example : To implement networked control systems, a test rig was built, based on an ARM9 embedded system. The forward channel is through a network and the communication protocol between the controller and the sensor is UDP. The kernel chip of the embedded board is ATMEL's AT91RM9200, which is a cost-effective, high-performance 32-bit RISC microcontroller for Ethernet- based embedded systems. A 10M/100M self-adaptive network controller is integrated in the chip and the chip also has a high computing performance and can work at speeds up to 180 MHz. 2-channel 16-bit high speed digital-analog (D/A) converters and 8-channel 16-bit high speed analog-digital (A/D) converters in the controller board provide I/O interfaces for the controlled plant.

In order to validate the proposed method, a servo motor control system which consists of a DC motor, peripheral equipment, speed sensors is considered. The model of the motor control plant at sampling period 0.04 second was identified to be

$$G(z^{-1}) = \frac{A(z^{-1})}{B(z^{-1})} =$$
$$\frac{0.3077 * z^{-1} - 0.05806 * z^{-2} - 0.09977 * z^{-3} - 0.09555 * z^{-4}}{1 - 0.3538 * z^{-1} - 0.3059 * z^{-2} - 0.2932 * z^{-3} + 0.006748 * z^{-4}}$$

The system can also be written as the state space form
with the following system matrices

$$A = \begin{bmatrix} 0.3538 & 0.3059 & 0.2932 & -0.0067 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.3077 & -0.0581 & -0.0998 & -0.0955 \end{bmatrix}$$

The matrices K and L were designed to be

$$K = \begin{bmatrix} -0.2468 & -0.3097 & -0.2931 & 0.0067 \end{bmatrix},$$

$$L = \begin{bmatrix} 3.0459 \\ 2.4805 \\ 2.0067 \\ -0.0515 \end{bmatrix}, M = \begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix}$$

which ensure the close-loop system without time delay is stable.

To illustrate the operation of the proposed networked predictive control scheme, two cases were considered:

a) Local control. There is no network in the closed-loop system, i.e., the output signal from the controller is directly connected to the plant. So, the network delay is zero. The design of matrices K, L and M has ensured that the closed-loop system is stable.

b) Intranet based control. In this case, the output signal was physically transmitted between two Intranet IP addresses 192.168.2.106 and 192.168.2.108 which were both located on the Chinese Academy of Sciences, Beijing. It was measured that the maximum network delay was 0.12 second. As the sampling period is 0.04 second, the upper bound N = 3. For k_t = 0, 1, 2, 3, all eigenvalues of matrices $\overline{A}$ (k) are stable and a common positive definite matrix P satisfying inequalities (13) was found using the LMI toolbox. That means the closed-loop system is stable with K ,L and M given above.

c) Internet based control. In this case, the output signal was transmitted between the same two Internet IP addresses 192.168.2.106 and 192.168.2.108 which were both located on the Chinese Academy of Sciences, Beijing. From [13], we get that the maximum network delay was measured to be 0.32 second .So we add 0.2 second random delay to simulate the internet delay. The sampling period was still 0.04 second. So, the upper bound N = 8. Similar to the Intranet control case, for k_t = 0, 1, 2, ..., 8, all eigenvalues of matrices $\Lambda(k_t)$ are stable and also a common positive definite matrix P satisfying inequalities (15) was derived using the LMI toolbox. That implies that the closed-loop system is stable for given K , L and M above.

To evaluate the performance of the networked predictive control scheme, one real-time simulation and one real-time experiment were carried out.

Case1: $u_t = u_{t|t} = K\hat{x}_{t|t}$

1) Real-time simulation. In this simulation, the servo motor plant to be controlled is represented by its model but the network was a real one. The simulations were performed using Matlab/Simulink/Real time Workshop. The real-time simulation diagram is shown in Figure1.The reference input is a square wave generated by the pulse generator block, which changes between 0 to 1000rpm with period 5s. The controller block Netctrl is the networked predictive controller. Blocks Recv and Send are the receiver and sender of the UDP communication protocol. All of them were designed using Matlab S-Functions. The simulated plant and the controller were executed in a ARM 9 embedded system. The real network (Intranet or Internet) was between UDP communication blocks Recv and Send in Figure 3.

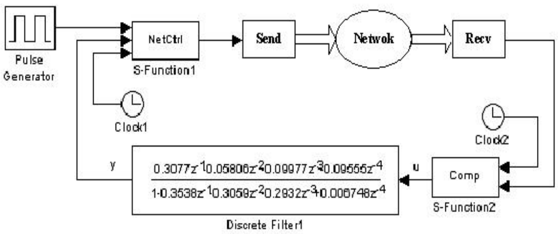

Figure 3: The diagram of the real-time simulation

Four real time simulations were conducted: local control (i.e., no network), Intranet based control without delay compensation, Internet based control with delay compensation and Intranet based control with delay compensation and The real-time simulation results are shown in Figures 5 and 6.The Internet based control without delay compensation was also conducted, but it was found that the system was no longer stable due to the large network delay, which was between 0.2-0.3s.

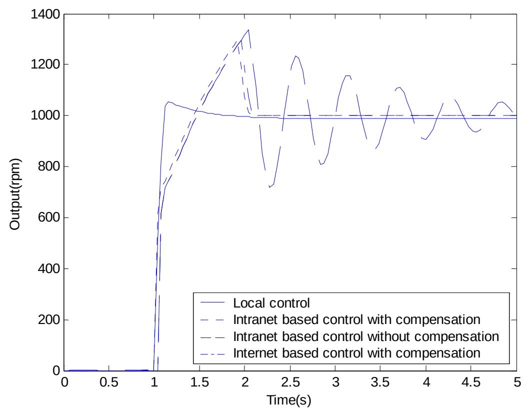

Figure 4: Outputs of servo plant (Simulation)

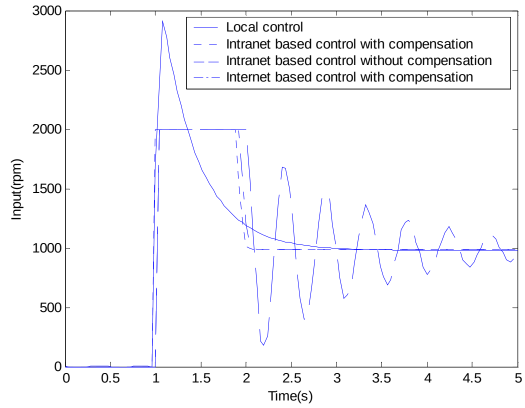

Figure5: Inputs of servo plant (Simulation)

2) Real-time experiment. The difference between the real-time simulation and real-time experiments is that the plant model of the servo motor in the real-time simulation is replaced by D/A block Dac and A/D block Adc and the real servo motor. The diagram of the real-time experiment is shown in Figure 7. The two blocks Dac and Adc were the driver of the A/D and D/A channels in the embedded system and were designed in Matlab S-Function. Similarly, four real-time experiments were made: local control (i.e., no network), Intranet based control without delay compensation, Intranet based control with delay compensation and Internet based control with delay compensation. The real-time experiment results are shown in Figures 8 and 9. Also, it was found that the Internet based control without delay compensation was unstable.

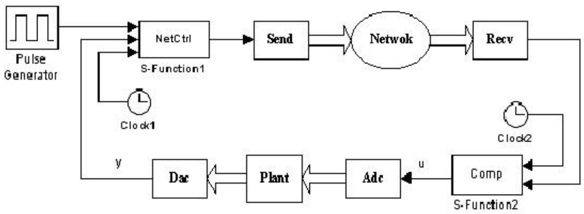

Figure6: The diagram of the real-time experiment

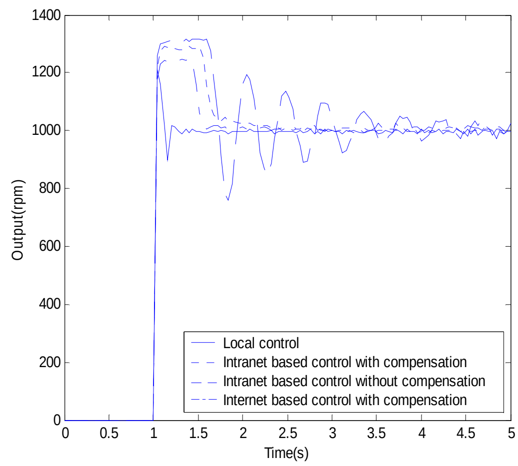

Figure 7: Outputs of servo plant (Experiment)

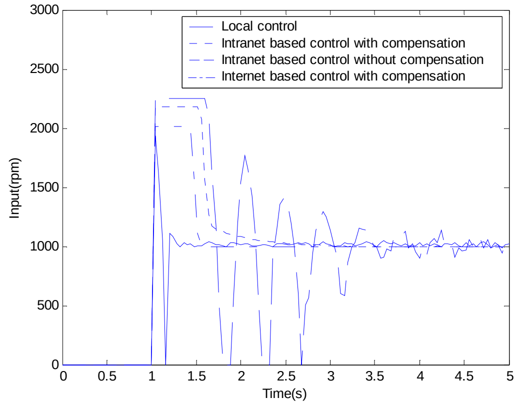

Figure 8: Inputs of servo plant (Experiment)

Case2: $u_t = u_{t|t-k} = K\hat{x}_{t|t-k}$

1) Real-time simulation

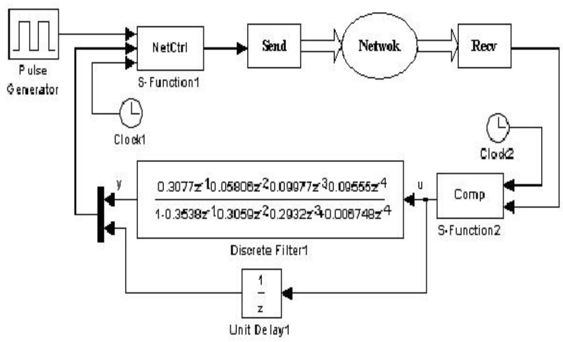

Figure 9: The diagram of the real-time simulation

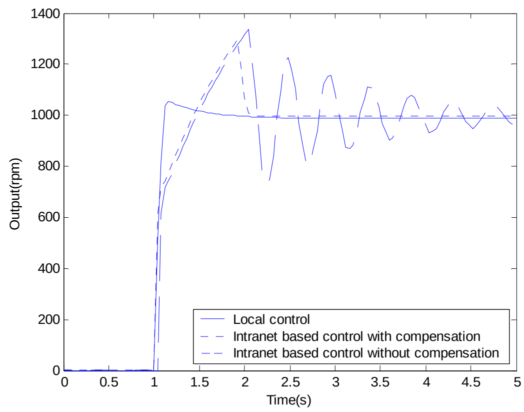

Figure 10: Outputs of servo plant (Simulation)

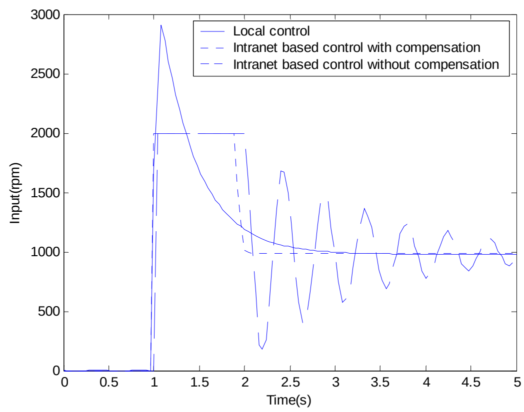

Figure 11: Inputs of servo plant (Simulation)

2) Real-time experiment.

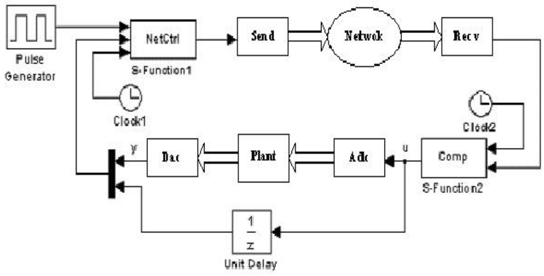

Figure12: The diagram of the real-time experiment

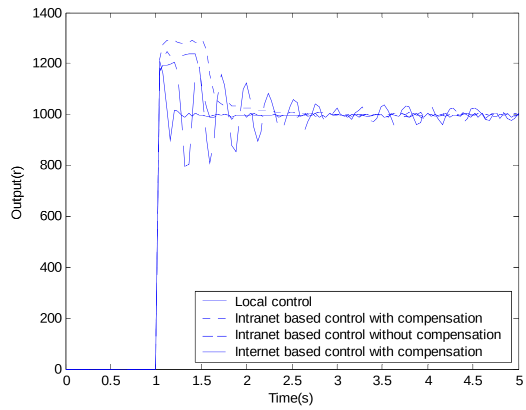

Figure 13: Outputs of servo plant (Experiment)

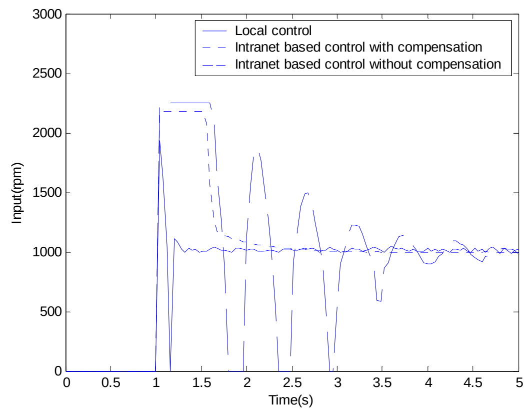

Figure 14: Inputs of servo plant (Experiment)

From the results of real-time simulations and real-time experiments, it is clear that the network transmission delay degrades the performance of NCS. But the networked predictive control scheme proposed in the paper can compensate for the network delay actively. Its performance is very close to that of the local control scheme (i.e., no network).

The results also shows the performances of two different ways to get u_t .We can see that case 2 is ideal, in which u_t is got from the plant directly. However, it is a pity that it is impossible in real-time experiment; but, we find that the effect of case 1 is very similar to case 2.

Although it is hard to make the model of the servo motor plant be exactly the same as the real practical one, the results of the real-time experiments are very similar to those of the real-time simulations.

5 CONCLUSIONS

A new networked predictive control scheme in a state-space form has been proposed for MIMO networked distributed control systems with random network delay and the stability of the closed loop networked predictive control systems has also been discussed in this paper. Based on the network feature of transmitting a set of data each time, the proposed networked predictive controller consists of the control prediction generator and the network delay compensator. The former provides a set of future control predictions to satisfy the system performance requirements. The latter compensates the random network transmission delay. Two important theorems on the stability of the closed-loop networked predictive control system have given the analytical stability criteria for different u_t, respectively.

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