K.Yu. Ermolenko, S.A. Vavilov

# Specification of control problems under uncertainty in finances

The attempts to apply classical methods of optimization based on the theory of optimal and adaptive control to realize the management of an investment portfolio very often tumble over serious problems. For instance the application of control theory as the stochastic version of dynamic programming approach implies the detailed information about the structure of factors in stochastic differential equations describing the dynamics of assets constituting portfolio. The latter information in contemporary financial markets seems hardly to be available. The methods of adaptive control theory are also not very often applicable because of the strongly nonstationary behavior of parameters of these or those modeling equations describing the dynamics of portfolio value.

Because of the aforesaid it is not surprising that the problem to create special control methods adapted to the investment portfolio management has long drawn the attention of researchers. Usually such methods imply the creation of control providing in a particular sense the positive dynamics of profit along with the minimization of quantitative and qualitative information about the structure of modeling equations. Moreover one of the most common models for assets pricing is the model of geometrical Brownian motion. Nevertheless when following this way to create the control of investment portfolio there arise a number of difficulties which may be formulated as follows. The heart of the matter is that the designing of control up till now has been based as a rule on the principles of self-financing strategy (see for instance [1]-[4]). The latter means that the purchase or sale of any assets automatically implies sale or purchase of a volume in the equivalent money terms of other assets constituting portfolio.

It is essential to note that realization of any circuit of management based on self-financing strategy implies (at least in terms of the literature available to the authors of the present work) the required number of assets in the portfolio significantly depends not only on the prices of struck bargains but also on the volatilities of corresponding assets.

The latter fact causes some inquires that seem to be an impediment in implementation of corresponding control systems. The heart of the matter is that for majority of liquid assets the values of their volatilities have strongly non-stationary and pronounced palpitating character. It makes the tracking of their values with arbitrary precision in real time hardly possible. It is also important to keep in mind the property of delay inherent in each control system based on continuous model of pricing and the necessity to realize discrete procedure for their implementation. In this connection it is clear that the occurrence of essential mistakes is possible while defining the amount of assets included in a portfolio. How significantly such errors can affect the ultimate goal of management to provide the profitableness of portfolio remains not clear.

The aforesaid makes reasonable to pose the problem of creating the management of portfolio with a feed-back control based only on the prices of struck bargains to provide in some sense portfolio profitableness on a certain time interval and within the framework of the pricing model corresponding to geometrical Brownian motion.

The main goal of the present study is to solve the problem under consideration within the framework of a management alternative to self-financing strategy. It implies the possibility to invest additional money resources from outside during the whole period of portfolio management. Moreover the release of cash as a result of trading allows its reinvestment to acquire new required assets.

Originally the idea to create control system on the basis of an approach alternative to self-financing strategy was formulated by one of the authors of the present work [5,6]. Later this idea was developed in a number of papers [7,8,9].

## References


1. J.M. Harrison and D.M. Kreps. 1979. Martingales and arbitrage in multiperiod securities markets. Journal of Economic Theory 20, 381-408.
2. Karatzas I., Lehoczky J.P., Shreve S.E. 1987. Optimal portfolio and consumption decisions for a “small investor” on a finite horizon. Journal of Control and Optimization 25, 1557-1586.
3. Back K., Pliska S. 1991. On the fundamental theorem of asset pricing with an infinite state space. Journal of Mathematical Economics 20, 1-33.
4. M. Frittelli and P. Lakner. 1995. Arbitrage and free lunch in a general financial market model: the fundamental theorem of asset pricing. Mathematical Finance, 89-92.
5. Vavilov S.A. 1998. On the stochastic control of investor portfolio when the price of security follows the Ornstein-Uhlenbeck process. In Abstracts of Communications of the International Conference "Asymptotic Methods in Probability and Mathematical Statistics", St. Petersburg, 298-302.
6. Vavilov S.A. 2001. On the probability models to control the investor portfolio. In the book: Asymptotic methods in probability and statistics with applications, Birkhauser, Boston-Basel-Berlin, pp. 535-546.
7. Vavilov S.A., Ermolenko K.Yu. 2003. Stochastic systems to control the investor portfolio – Bulletin of Saint-Petersburg State University, Series 5 “Economy”, Issue 3, 113-122 (in Russian).
8. Vavilov S.A., Ermolenko K.Yu. 2004. The procedure to smooth stock prices without using parameters adjusted to the historical data – Bulletin of Saint-Petersburg State University, Series 5 “Economy”, Issue 2, 97-106 (in Russian).
9. Vavilov S.A., Ermolenko K.Yu. 2005. The method to determine one integral characteristic for volatilities in the problem of portfolio management – Bulletin of Saint-Petersburg State University, Series 5 “Economy”, Issue 1, 114-124 (in Russian).