Negative differential conductivity of a system due to development of Turing structure

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Abstract— **Formation of a large amplitude Turing pattern may be accompanied by appearance of the negative differential conductance in a self-organized system. The effect is observed at studying a two-component reaction-diffusion model introduced earlier to interpret the occurrence of pattern formation in semiconductor-gas discharge devices. The revealed autocatalytic process can initiate further complicated scenarios of self-organization of a system.**

I. Introduction

It is well known that conducting systems can manifest multi-valued current-voltage characteristics, which is specified as the appearance of the negative differential conductance (NDC). In principle, the NDC can be revealed from measurement of the voltage drop on electrodes of a system while varying its current. (Or, vice versa, by varying the voltage, and measuring the current). At the presence of NDC, the homogeneous state of a spatially extended system may become unstable, while there may grow spatially non-homogeneous fluctuations. This results in the appearance of a dissipative structure. In semiconductor electronics, classic examples of dissipative structures are formation of current filaments in devices that manifest the current voltage characteristic (CVC) of *S*-type and domains of high electric fields in samples that show the *N*-type CVC [1-3]. Notice that the decrease in the voltage drop on a system's electrodes under the current increase gives the evidence of the S-type behavior of NDC.

In the present study, we consider an example, where a dissipative structure itself is responsible for formation of the NDC of a system. In such a case, the "primary" dissipative structure may be responsible for further development of the self-organized system. This result is obtained for a two-component reaction-diffusion model where formation of primary dissipative structures is due to the Turing instability of the spatially homogeneous state. The model has been introduced earlier [4] to interpret the experimentally observed phenomenon of formation of spatial patterns in dc-driven planar semiconductor-gas discharge systems [4,5].

A schematic representation of the pattern-forming device is shown in Fig. 1. Its main parts are the semiconductor plate and the gas-filled gap, which thicknesses are d_s and d, correspondingly. Both sides of this two-layered structure are covered with thin-film electrodes that are transparent with respect to light. The device is fed by a d.c. voltage source. The value of gas-discharge current, which is along the *z* axis, is controlled with the semiconductor electrode. Its resistance can be varied by applying the irradiation with infrared light.

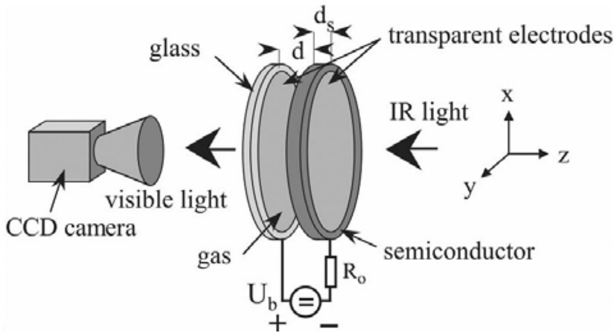

Fig. 1. Semiconductor-discharge gap pattern-forming device (schematically).

The spatial distribution of current in the device can be evaluated by observing the corresponding distribution of the gas glow in the gas-discharge gap. This can be done by applying a CCD camera that is sensitive in the visible range of light. For further details, see, e.g., [4,5].

II. The model studied

To describe formation of patterns in the device represented in Fig. 1, next equations were introduced in [4]:

$$\frac{\partial U}{\partial t} = \frac{U_0 - \gamma \overline{N} - U}{\tau_U} - cNU + D_U \Delta U \qquad (1)$$

$$\frac{\partial N}{\partial t} = -\frac{N}{\tau_N} + NU\left[a + b\left(\frac{N}{N+N^*}\right)^2\right] + D_N \Delta N \qquad (2)$$

$$\overline{N} = \frac{1}{S}\int_S N dS ,$$

where variables U and N in (1,2) are the voltage drop on the gas discharge gap, and density of free charge carriers in the gap, correspondingly; a, b, c and γ are coefficients; S is the square of the planar system; $\overline{N}$ - density of carriers averaged over the plane of the system; U_0 is the voltage feeding the device.

The first equation describes the charging the capacity of the discharge gap from the voltage source U_0, and its discharging via the presence of free carriers in the gap. The charging process proceeds with the time constant τ_U, which value is dependent on the resistivity of the planar

The work is supported by President Grant of Russia for Scientific Schools (project SSch-5920.2006.2) and the Russian Foundation for Basic Research (project 07-02-00168-a).

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semiconductor electrode. The term $\gamma \bar{N}$ takes into account the global negative feedback that exists in the physical pattern-forming system, or can be incorporated intentionally by adding a resistor R_0 into the electrical circuit feeding the device, see Fig. 1.

The second equation describes dynamics of charge carriers in the gap. The rate of their generation is controlled by the second term. The term in quadratic brackets takes into account the efficiency of the autocatalytic process of carriers' generation. Other conditions being equal, it can be regulated by changing the value of parameter N^*. Characteristic time τ_N defines the rate of carriers decay at the absence of their generation. Last terms in equations describe diffusive spreading of variables in the plane. We point out that the variable N activates the formation of spatial structures in the system, while U plays the inhibiting role in this process.

III. Some properties of model

We are interested in patterned states of the non-linear equations (1,2) on two-dimensional domain x,y. In general, they can be found only by numerical calculations. However, some important features of the model may be clarified when finding stationary homogeneous solutions of equations. At the absence of the external load ($\gamma = 0$), these states are stationary solutions to the next equations:

$$\frac{\partial U}{\partial t} = \frac{U_0 - U}{\tau_U} - cNU \qquad (3)$$

$$\frac{\partial N}{\partial t} = -\frac{N}{\tau_N} + NU\left[a + b\left(\frac{N}{N+N^*}\right)^2\right] \qquad (4)$$

As an important characteristic of such states, we consider values of N_S as dependent on the feeding voltage U_0. Examples of corresponding dependences obtained for four values of τ_U are shown in Fig. 2. The curves are calculated at the next set of parameters of (3,4): $\tau_N = 10^{-3}$ sec; $a = 0.83$, $b = 0.2$, $c = 2\times10^{-4}$ cm^3/sec; $N^* = 5\times10^4$ cm^{-3}. Remark that in a real device, τ_U may be controlled optically, by varying the intensity of infrared light J used to irradiate the semiconductor electrode, see Fig. 1. Increase in J is accompanied by the decrease of τ_U.

The data of Fig. 2 illustrate that varying the value of τ_U can qualitatively change the dependence $N_S = f(U_0)$: Increase in τ_U can initiate transition from the S-type curve (where voltage on electrodes of the system decreases with increase of the density of carriers in the gap) to a monotonic one. In other words, increase in τ_U is followed by transition from the S-type negative differential conductance state to a state without the NDC.

Notice that two-component models that are close to (1,2) have been intensively studied in the theory of self-organized systems, see, e.g., [6-9]. These models contain terms responsible for the autocatalytic reproduction of one variable, and the diffusion coupling of the two variables. Among relatively simple models, we point out a so called brusselator [9]. It has been introduced and studied by I. Prigogine and his collaborators in Brussel. The specific feature of brusselator is the monotonic dependence of the global "transport" curve – similar to the dependencies observed in our case for large τ_U values, see Fig. 2.

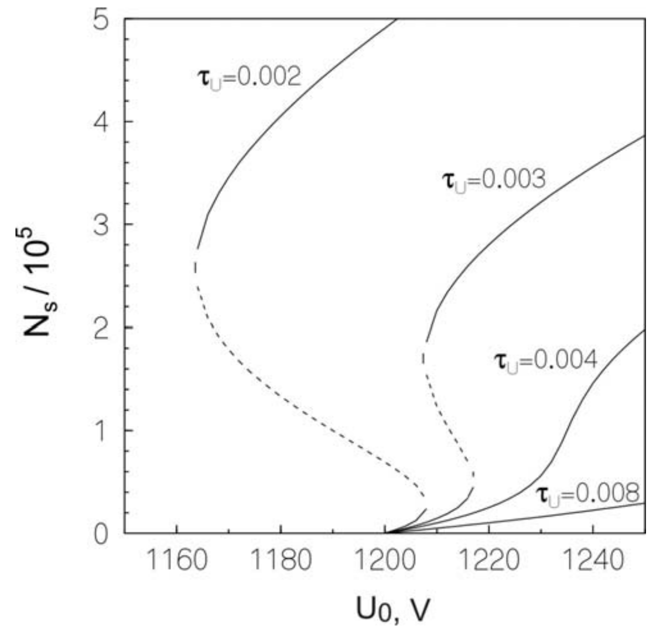

Fig. 2. Dependencies of stationary density of carriers on the feeding voltage for different values of parameter τ_U. For the other parameters of calculations, see the text.

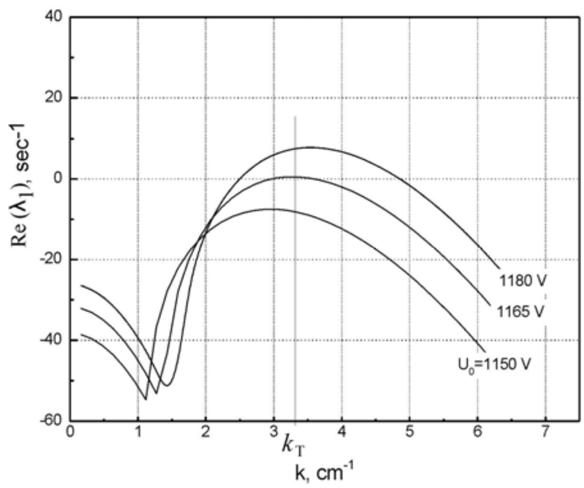

Fig. 3. Real part of increment of small perturbations of stationary solutions as dependent on wave vector. Data are calculated for the one dimensional version of (1,2). For values of parameters, see the text.

In the present paper, we will deal with the brusselator-like version of the model (1,2). That is, we are interested in the range of parameters of equations where the stationary homogeneous state of the system does not exemplify the NDC. As is well-known, such state is stable against perturbations of transverse (in the plane *x,y*) wave vectors $k \approx 0$; that is, the amplitude of the spatially homogeneous state can not spontaneously change.

It can be shown, however, that, similar to brusselator, the stationary homogeneous solution of (1,2) may become unstable against periodic in space perturbations when the spatial coupling of variables is included. This is demonstrated in Fig. 3, where a set of spatial modes calculated at different values of the feeding voltage U_0 is shown. The data are obtained for the one-dimensional version of (1,2) at $\gamma = 0$. The next set of parameters is used in calculations: $\tau_N = 10^{-3}$ sec; $\tau_U = 10^{-2}$ sec; $a = 1.0$; $b = 0.4$; $c = 1.64\times10^{-4}$ cm^3/sec; $N^* = 1.5\times10^5$ cm^{-3}; $D_U = 0.625$ cm^2/sec; $D_N = 0.045$ cm^2/sec.

For the example presented, the system remains stable at low U_0, while the modes decay for perturbations of all spatial scales. Increasing U_0 up to some, critical, value results in appearance of a perturbation which decay is zero. In the case considered, this occurs for the perturbation of wave vector k_T at $U_0 \approx 1165$ V. At a further increase in U_0, there appear growing modes, which evidences the instability of the homogeneous state of the system against small amplitude perturbations.

IV. Formation of Turing structures

The origin of instability that is described in the previous Section, is essentially defined by the diffusion process. The diffusion mechanism of instabilities, for the case of two variables, was treated for the first time in [10]. It was demonstrated in the cited paper that it can give a spatially periodic stationary structure. Now it is referred to as the Turing mechanism of pattern formation.

The numerical investigation of the model (1,2) on two-dimensional domain [4] supports the conclusion of the previous Section that, at proper set of parameters, a stationary, spatially homogeneous solution becomes unstable against small-amplitude perturbations when the feeding voltage U_0 reaches some critical value U_0^{crit}. In general, the instability gives rise to the growth of the hexagonal pattern.

Formation of a hexagonal pattern on the two-dimensional domain is known to be characterized by the hysteretic behavior. This is also observed in the considered case: The decay of the hexagon occurs at a value of U_0 that is lower than U_0^{crit}. We notice that this regularity is in correspondence with experimental results obtained at studying the semiconductor-gas discharge pattern-forming system [4, 11] represented in Fig. 1. Remark that the emerging pattern there is the hexagonal arrangement of current filaments.

The possibility of current filaments to exist in a sub-critical domain means that they can *co-exist* with the homogeneous background. The number of filaments (correspondently, of N maxima in calculations) inside the active area of a system may vary and depends on the system's history. In essence, these objects are dissipative solitons (DS) [12,13] (they are also referred to as autosolitons [14]) that can exist stable in a non-equilibrium system.

Among other remarkable properties, DSs are known to initiate the appearance of new DSs when control parameters are varied. This may occur either via the division of primary DSs [14-16], or via the so called self-completion process [14]. In the last scenario, new-born DSs are generated at the neighborhood of existing ones. For a two-dimensional system, this phenomenon has been observed in numerical analysis of (1,2) and in the experimental investigation of the semiconductor-gas discharge system [4].

V. Negative differential conductance in the course of formation of a structure

Investigation of the model (1,2) gives the evidence that formation of a spatially extended pattern in the course of the self-completion process can lead to the NDC of the whole system. This conclusion follows from the result of the next calculations which stages are illustrated by Fig. 4 and 5. Data of Fig. 4 show the average density of carriers as dependent on the voltage drop on the system U_S (this is the difference between the applied voltage U_0 and the voltage drop on the load modeled by the term $\gamma \overline{N}$). These data have been obtained as follows. A solution of the system that contains one DS within the analyzed active domain has been used as the initial one. This is the stationary state, where the DS coexists with the homogeneous background, see Fig. 5 a. In the plane ($\overline{N}$, U_S), this state corresponds to the point (a) on the curve of Fig. 4. Then, the gradual increase in the feeding voltage U_0 is applied in the calculation procedure. This leads to the nearly linear increase in $\overline{N}$, which is observed up to the voltage $U_{self-compl}$, Fig. 4. Reaching this critical voltage changes qualitatively the state of the system. The initial DS stimulates the birth of new DSs in its neighborhood via the self-completion process, see Fig. 5 b. The process repeats, which is followed by spreading of the hexagonal pattern over all the active area of the system, stages **b**,**c**,**d** in Fig. 5.

The amplitude of stationary DSs N_{max} is essentially higher than that for the homogeneous background. That is, the birth of new DSs is accompanied by an increase in the total quantity of activator N. At the absence of the "external" load (that is, at $\gamma = 0$) the self-completion of the hexagonal pattern results in the transition of the system along the vertical line (**a'**-**e**) in Fig. 4. This process lasts till all the active area of the system becomes filled with the pattern.

In the case $\gamma \neq 0$, the growth of the hexagon, which gives the increase in the overall quantity of activator, is accompanied by diminishing the voltage drop on the structure, because larger and larger part of the applied voltage drops on the load. That is, the NDC is revealed in the global transport characteristics of the system, which is reflected by the falling of voltage U_S as the pattern grows (Fig. 4). When all the area is filled with the hexagon (Fig. 5 **d**), the characteristic becomes again linear, domain (**d**) in the curve of Fig. 4. We remark also that the details of development of the system depend on the rate of increase in the feeding voltage. Other conditions being equal, at diminishing the rate of increasing U_0 in the calculation procedure is followed by observing a somewhat less effective value of $U_{self-compl}$. In this case, the overall characteristic can somewhat shift to lower voltages, as represented by the sequence of round points in Fig. 4.

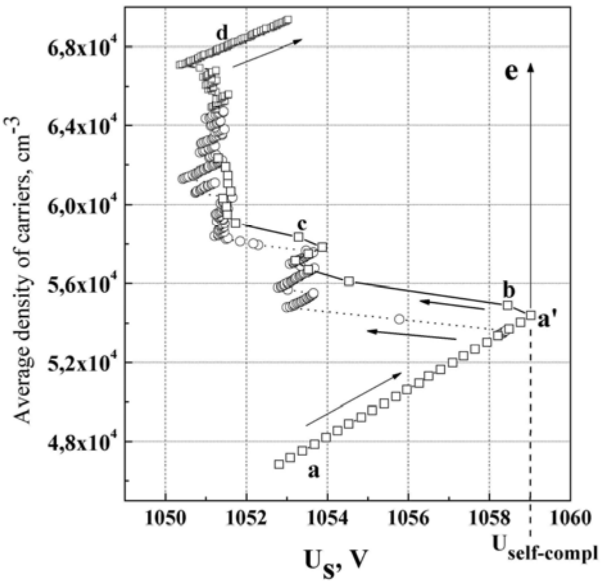

Fig. 4. Dependences of average density of carriers on the voltage drop on the structure U_S. The points on the plot are obtained at $\gamma = 5\times10^{-3}$. The other parameters of calculations are the same as those used to obtain data of Fig. 3. Calculation is done for quadratic area of linear dimension L=3.5 cm. Two sets of data that are shown with square and round points are obtained at somewhat different rates of increasing the parameter U_0.

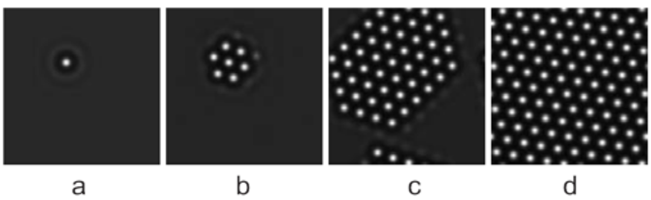

Fig. 5. A realization of the self-completion scenario of the hexagonal structure. Shown is the sequence of spatial distributions of the activator N that is observed while going along the global transport characteristic of Fig. 4 marked with square points. Letters below the pictures comply with stages labeled by corresponding letters on the curve of Fig. 4.

VI. Discussion and Conclusions

In case studied in the present work, we deal with two mechanisms of differential negative resistance. The first one exists in the spatially homogeneous state (in the reference state). It is due to the second term in the square brackets of Eq. (2), which models the non-linear transport characteristic of the gas discharge gap. This NDC is the “internal” one, that is, it does not appear in the global transport characteristic N (U_S) for the reference (spatially homogeneous) state. Together with the diffusion of components, it is responsible for formation of Turing patterns. Contrary to this mechanism, the second one is realized when a large-amplitude Turing pattern develops. The growth of the pattern leads to the strong non-linearity in the global transport characteristic of the system, which is due to the essential increase of quantity of the activator N. At presence of the global load, the NDC of the system is then detected.

The data of Figs. 4,5 present also another interesting feature of the pattern growth via the self-comletion scenario: Increase in number of DSs that constitute the pattern is followed by a decrease of the voltage needed to initiate a further growth of the pattern in space. Indeed, as follows from these Figures, the spreading of the hexagon in space is observed at values of the voltage drop on the system that are lower than the critical voltage needed to start the self-completion process from a solitary DS. So, the growing pattern renders the autocatalytic action on its further development. This phenomenon seems to be related with a coherent perturbation of the homogeneous surrounding by the ordered ensemble of DSs, which is more efficient than the action of a lonely DS.

Finally, the presence of the global NDC (that is, the NDC that can be measured on electrodes of a system) is known to initiate the oscillatory dynamic of systems. In the case of the spatially extended system considered here, interesting scenarios of further spatio-temporal self-organization can be realized. As an example, there can appear oscillatory dynamic of the system, where Turing patterns exist in the bursting mode. According to preliminary experimental data, such a regime of self-organization may be observed in the planar cryogenic semiconductor-gas discharge device described in [4,3].

Acknowledgment

The author thanks Dr. Yu. A. Logvin (Metrophotonics, Canada) for help in obtaining data presented in the work, and Dr. A. V. Gorbatyuk (Ioffe Physico-Technical Institute, St. Petersburg) for valuable discussions.

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