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FAULT DIAGNOSIS SCHEME FOR NONLINEAR STOCHASTIC SYSTEMS WITH TIME-VARYING FAULT: APPLICATION TO THE RIGID SPACECRAFT CONTROL

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Abstract
This paper studies the problem of the fault estimation for a class of time-varying faults using output probability density function (PDF). In particular, the spacecraft control system is studied. First, the attitude control of the nonlinear model with uncertainties is given. Then, the measured output is viewed as a stochastic process and its PDF is modeled, which leads to a deterministic dynamical model including nonlinearities and uncertainties. A new adaptive fault diagnosis algorithm is proposed to improve the performance of the fault estimation. The proposed algorithm contains both the proportional and the integral term. The proportional term can improve the speed of the fault estimation, while the integral term can eliminate estimation error. Then, based on the linear matrix inequality (LMI) technique, a feasible algorithm is explored to find the designed parameters. Finally, simulation results of the spacecraft are given to show the efficiency of the proposed approach.

Key words
Attitude control, adaptive fault estimation, LMI, PDF.

1 Introduction

Increased productivity requirements and stringent performance specifications lead to more demanding operating conditions of many engineering systems. Such conditions increase the possibility of system failures. Sensor, actuator or plant failures may drastically change the system behavior, resulting in degradation or even instability. As an important aspect for practical processes, such as spacecraft, large-scale chemical engineering processes, biochemical processes and biodiesel processes, the safety and reliability problem of control system has long been investigated [Ruiyun *et al.*, 2011], [Yuhai, Ruiyun and Gang, 2012] and [Liying *et al.*, 2012]. In order to improve efficiency, the reliability can be achieved by the fault tolerant control (FTC), which relies on early detection of faults, using fault detection and isolation (FDI) procedures. So FDI has become an attractive topic and received considerable attention during the past two decades. In such a way, FTC and FDI belong to the recent control theoretic investigation mainstream, dealing with certain abrupt changes in models, similarly to the multi-agent systems, see [Proskurnikov, 2012] and references within there. Most conventional fault detection approaches are designed for nonlinear stochastic system, and they are based on some inherent mathematical redundancy of the combination of the system and the observer. Stochastic methods are widely spread to handle the control and estimation under uncertainty, see [Rigatos, 2012; Dolinsky and Čelikovský, 2012] and references within there. The commonly used nonlinear stochastic system for FDI is [Guo and Wang, 2005; Wand and Daley, 1996; Jian, Staroswiecki and Cocquempot, 2006; Zhang, Guoi and Wang, 2006]

$$\dot{x}(t) = Ax(t) + g\left(x(t), t\right) + Bu(t) + Ef^c(t) \quad (1)$$
$$y(t) = r\left(x(t), u(t), \xi(t), f^c(t)\right) \quad (2)$$

where $x(t)$ is the state vector, $u(t)$ is the known input vector, $y(t)$ is the output vector measured by sensors, $g\left(x(t), t\right)$ is a continuous nonlinear vector function. Further, $f^c(t)$ is the unknown fault input vector, which is also known as the actuator fault and $\xi(t)$ is the output noise. Various FDI techniques are designed for the system (1) and (2) including filter-based or observer-based approaches. Up to now, most approaches concentrate on Gaussian systems. In fact, some processes exhibit asymmetric non-Gaussian distribution, the expectation of the traditional Kalman fil-

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tering approach is obviously insufficient to characterizing such processes and hence the probability density function (PDF) approach is needed. The PDF approach is actually a shape control method. To approximate a kind of distribution, one way is to use static-based approach, such as Monte-Carlo approach or particle filter approach, where Bayesian lemma and likelihood method are used. Another way is to use function or functional approach, such as spline approach [Wang and Lin, 2000], [Wang, 2001], [Guo and Wang, 2004] and [Guo and Wang, 2005] where B-spline functions are used.

In references [Wang and Lin, 2000], [Wang, 2001], [Guo and Wang, 2004] and [Guo and Wang, 2005], a class of general stochastic system has been investigated, an output PDF approach via B-spline functions has been presented. The B-spline bases represent the space information of an output distribution while the weighting functions reflect its time-varying information. The output PDF approach transfers the corresponding stochastic process to a deterministic dynamical system, and hence the corresponding stochastic problem is transferred to a deterministic one.

In this paper, the problem of the fault estimation for the spacecraft using angular rate supplied by momentum wheel actuators as inputs is investigated. A rigid spacecraft in general is controlled by three independent actuators and it is well known that three momentum wheels can be used to accomplish arbitrary reorientation maneuvers of the spacecraft using smooth feedback control law. Three reaction wheels are considered as input actuators. Fault scenarios of reaction wheel usually include high motor frictions; drop of bus voltage, unusual motor disturbances and unexpected current variations, these anomalies can be conceptualized as form of time-varying faults and constant faults.

Obviously, even before our research to be presented here, many researchers have paid more attention to adaptive fault diagnosis observer using PDF as well. More specifically, the main limitations in the use of this conventional approach are twofold. First, performance requirements of the fault estimation, i.e., speed and accuracy. Second, existence conditions of adaptive fault diagnosis observer have not been given explicitly in existing works, which adds difficulties for the design of adaptive fault diagnosis observer. Therefore, investigating an effective solutions to overcome the above difficulties is necessary, and thus motivates the research presented in this paper.

More specifically, our objective is to analyze the model-based fault estimation scheme and to develop a general framework for a novel adaptive fault diagnosis approach using PDF. This extends earlier results of fault estimation using adaptive fault diagnosis algorithm [Nguyen and Čelikovský, 2012] in order to be applicable to the rigid spacecraft control problem analyzed in the current paper as well. In such a way, the current paper contributes to the important interdicsiplinary area between physics and control being the flight and aerospace applications, see [Amelin, 2012] and references within there.

The rest of this paper is organized as follows. Section 2 gives system description for the spacecraft attitude control system with actuator fault and background on the conventional adaptive fault diagnosis approach using output PDF. The novel adaptive algorithm for actuator fault estimation is presented in Section 3. Simulation results in Section 4 show the effectiveness of the proposed approach. The concluding remarks are given in the final section. In the following, the notation $\|\cdot\|$ denotes the Euclidean norm of the vector on $\mathbb{R}^3$. The dimensions of the matrices, if not stated explicitly, are assumed to be compatible. The identity and zero matrices are denoted by I and 0, respectively, with appropriate dimensions.

2 Mathematical Model of the Spacecraft and Problem Formulation

The equations describing the attitude control problem are basically those of a rotating rigid body with extra terms describing the effect of the control torques. They therefore consist of kinematic equations relating the angular position with the angular velocity, and dynamic equations describing the evolution of angular velocity.

2.1 Kinematic Equations

The orientation of the spacecraft can be specified using various parametrization of the special orthogonal group $SO(3)$. We describe the angular position by a rotation matrix $\mathbf{R}$. $\mathbf{R}$ transforms a inertially fixed set of orthonormal axes $x_1,\ x_2,\ x_3$, denoted I, into a set of orthonormal axes r_1, r_2, r_3, denoted Q, of the same orientation, and fixed in the spacecraft, with the origin at the center of mass. We have

$$Rx_i = r_i, \quad i = 1, 2, 3.$$

Thus, the equation describing the orientation of the rigid body is

$$\dot{R} = S(\omega)R, \tag{3}$$

where ω is the absolute angular velocity of the spacecraft measured in Q and $S(\omega)$ is a 3×3-matrix defined by:

$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & \omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}.$$

Here $\omega = \sum_{i=1}^{3} \omega_i r_i$ and the operator $\mathbf{S}$ is related to the vector product in $\mathbb{R}^3$ via

$$S(a)b = b \times a.$$

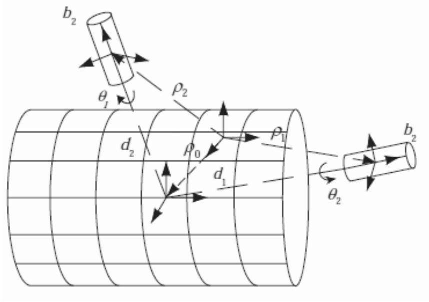

Figure 1: Spacecraft system

It is easily shown that Eq. (3) may be written as

$$R = RS(R^t\omega). \tag{4}$$

Then, the rows v of R satisfy $\dot{v} = R^t\omega \times v$, the usual equation for the evolution of a vector v rotating with angular velocity $R^t\omega$ in the inertial axes I. The angular position may be described locally by three angles ψ, θ, ϕ, which represent consecutive close rotation about $r_1,\ r_2,\ r_3$. Setting r_i to be the standard i-the basis vector in R^3 we obtain the kinematic equations as follows

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} = \begin{bmatrix}\dot{\phi}\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}1 & 0 & 0\\ 0 & cos\phi & sin\phi\\ 0 & -sin\phi & cos\phi\end{bmatrix}\begin{bmatrix}0\\ \dot{\theta}\\ 0\end{bmatrix} + \begin{bmatrix}1 & 0 & 0\\ 0 & cos\phi & sin\phi\\ 0 & -sin\phi & cos\phi\end{bmatrix}\begin{bmatrix}cos\theta & 0 & -sin\theta\\ 0 & 1 & 0\\ sin\theta & 0 & cos\theta\end{bmatrix}\begin{bmatrix}0\\ 0\\ \dot{\psi}\end{bmatrix}.$$

Therefore,

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}1 & sin\phi.tan\theta & cos\phi.tan\theta\\ 0 & cos\phi & -sin\phi\\ 0 & sin\phi/cos\theta & cos\phi/cos\theta\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}. \tag{5}$$

Although these angles do not coincide with the usual definition of Euler angles, they give rise to a convenient set of linearized equations. Clearly, the Euler angles are limited to the range:
$-\pi \leq \phi \leq \pi, -\pi/2 < \theta < \pi/2, -\pi \leq \psi \leq \pi$.

2.2 Dynamic Equations

Let J be the inertia matrix of the spacecraft, and b_i, the axes about which the corresponding control torque $\|b_i\|u_i$ is applied by means of opposing pairs of gas jets. Further, m will designate the number of control torques. The following closed set of equations describe the attitude control problem.

$$\dot{R} = S(\omega)R,$$
$$J\dot{\omega} = S(\omega)J\omega + \sum_{i=1}^{3} b_i u_i. \tag{6}$$

2.3 Actuator Fault

Defining the state vector as $x = [\phi\ \theta\ \psi\ \omega_1\ \omega_2\ \omega_3]^T$, the attitude dynamics of the spacecraft with actuator fault is summarized by combining (5) and (6)

$$\dot{x}(t) = A_k x(t) + g_k\left(x(t), t\right) + B_k u_k(t) + E_k f(t) \tag{7}$$

$$y(t) = r\left(x(t), u(t), \xi(t), f_k^c(t)\right) \tag{8}$$

where $u_k = [u_1\ u_2\ u_3]^T$ denotes control torques that are applied to the three axes, $B_k = E_k = \left[0_{3\times3}\ J^{-1}\right]^T$,

$$A_k = \begin{bmatrix} 0 & 0 & \omega_0\ 1\ 0\ 0\\ 0 & 0 & 0\ \ 0\ 1\ 0\\ -\omega_0 & 0 & 0\ \ 0\ 0\ 1\\ \frac{-3\omega_0^2(J_2-J_3)}{J_1} & 0 & 0\ \ 0\ 0\ 0\\ 0 & \frac{-3\omega_0^2(J_1-J_3)}{J_2} & 0\ \ 0\ 0\ 0\\ 0 & 0 & 0\ \ 0\ 0\ 0 \end{bmatrix},$$

$$g_k(x(t), t) = \begin{bmatrix} 0\\ -\omega_0\\ 0\\ \frac{(J_3-J_2)\omega_2\omega_3}{J_1}\\ \frac{(J_1-J_3)\omega_1\omega_3}{J_2}\\ \frac{(J_2-J_1)\omega_1\omega_2}{J_3} \end{bmatrix}, \tag{9}$$

$J = [J_1,\ J_2,\ J_3]^T$ stands for principal moments of inertia in roll, pitch and yaw axes. Further $y(t)$ is the measured output, $\xi(t)$ is the output noise and $g_k(x(t), t)$ is a continuous nonlinear function satisfying Lipschitz condition. More specifically, let us assume that there exists a known Lipschitz constant $L_g > 0$ such that

$$\|g_k\left(x_1(t), t\right) - g_k\left(x_2(t), t\right)\| \leq L_g\|x_1(t) - x_2(t)\|$$

for any $x_1(t)$ and $x_2(t)$. Moreover, $f_k^c(t)$ represents the actuator faults. The failure $f_k^c(t) = \beta(t - t_f)f(t)$ can be treated as an additive signal, where the function $\beta(t - t_f)$ is given by

$$\beta(t - t_f) = \begin{cases} 0, & t \leq t_f\\ 1, & t > t_f \end{cases} \tag{10}$$

where t_f is the time of fault occurring.

Let us note that only the continuous time-varying actuator fault is addressed in this paper. That is, $f_k^c(t)$ is zero prior to the failure time $(t \leq t_f)$ and it is equal to $f(t)$ after the failure occurs $(t > t_f)$. The fault effect is modelled by a "fault pattern", described by the distribution matrix E_k and a "fault parameter" $f(t)$, which can be time varying, and is supposed to be norm bounded. Throughout the rest of the paper $f(t)$ will stand for the above mentioned fault parameter, also referred to as the fault vector.

Assumption 1. *It is assumed that the fault vector and its time derivative are bounded, i.e., there exist two constants* $f_0 \in \mathbb{R}^+$ *and* $f_1 \in \mathbb{R}^+$ *such that:* $\|f(t)\| \leq f_0$, $\|\dot{f}(t)\| \leq f_1$.

2.4 Conventional Adaptive Fault Diagnosis Observer Using PDF

Suppose that the measured output satisfies $y(t) \in [a, b]$. Based on the statistical information of sample data, the distribution function of the output sample can be obtained, and the corresponding probability density function (PDF) can be further studied. Of course, the output distribution law is usually complicated, which often results in the complexity of the output PDF. To obtain the output PDF, the B-spline approximation technique is often used. The conditional probability of output $y(t)$ lying inside $[a,\ \xi]$, $\xi \leq b$ is defined by $P\{a \leq y(t) \leq \xi\}$, which can be expressed as

$$P\{a \leq y(t) \leq \xi\} = \int_a^{\xi} \gamma(z, u(t), f(t))dz, \quad (11)$$

where $\gamma(z, u(t), f(t))$ represents the output PDF and z is the variable defined on $[a,\ b]$. Similar to [Guo and Wang, 2004] and [Guo and Wang, 2005], we use the following square root B-spline approximation technique to model $\gamma(z, u(t), f(t))$:

$$\sqrt{\gamma(z, u(t), f(t))} = \sum_{i=1}^{n} v_i\left(u(t), f(t)\right) b_i(z) \quad (12)$$

where $b_i(z)$ $(i = 1,\ 2,\ \ldots,\ n)$ are pre-specified basis functions defined on $[a,\ b]$ and $v_i(u(t), f(t))$ $(i = 1, 2, \ldots, n)$ are corresponding weights of such an function. Let

$$B(z) = [b_1(z)\ \ b_2(z) \cdots b_{n-1}(z)]$$
$$V\left(u(t),\ f(t)\right) = \left[v_1\left(u(t), f(t)\right) \cdots v_{n-1}\left(u(t), f(t)\right)\right]^T,$$

and let

$$\Lambda_1 = \int_a^b B(z)^T B(z)dz, \quad \Lambda_2 = \int_a^b B(z)^T b_n(z)dz,$$
$$\Lambda_3 = \int_a^b b_n^2(z)dz \neq 0.$$

Then, it can be easily verified that (12) can be rewritten as (see [Guo and Wang, 2004] for details)

$$\sqrt{\gamma(z, u(t), f(t))} = B(z)V(t) + h\left(V(t)\right) b_n(z), \quad (13)$$

where

$$h\left(V(t)\right) = \frac{\sqrt{\Lambda_3 - V^T(t)\Lambda_0 V(t)}}{\Lambda_3} \quad (14)$$

and $\Lambda_0 = \Lambda_1\Lambda_3 - \Lambda_2\Lambda_2^T$. Here, $V\left(u(t), f(t)\right)$ has been abbreviated as $V(t)$. Then the conventional Lipschitz condition is assumed to be satisfied for $h\left(V(t)\right)$ in (13) within its operated region, i.e., for any $V_1(t)$ and $V_2(t)$, there exists a known Lipschitz constant $L_h > 0$ satisfying

$$\|h\left(V_1(t)\right)\| - \|h\left(V_2(t)\right)\| \leq L_h\|V_1(t) - V_2(t)\|.$$

The output PDF model is set up if $V(t)$ is modeled. Suppose that $V(t)$ satisfies

$$V(t) = Cx(t), \quad (15)$$

where C is known matrix. Then system (7-8) can be written as the follows model

$$\dot{x}(t) = A_k x(t) + g_k\left(x(t), t\right) + B_k u_k(t) + E_k f(t) \quad (16)$$
$$V(t) = Cx(t) \quad (17)$$

where $V(t)$ is the weight vector.

In order to diagnose the fault, the conventional adaptive fault diagnosis observer is constructed as (see [Guo and Wang, 2005] for details)

$$\dot{\hat{x}}(t) = A_k\hat{x}(t) + g_k\left(\hat{x}(t), t\right) \quad (18)$$
$$+ B_k u_k(t) + E_k\hat{f}(t) + L\varepsilon(t) \quad (19)$$
$$\hat{V}(t) = C\hat{x}(t) \quad (20)$$
$$\varepsilon(t) = \int_a^b q(z) \quad (21)$$
$$\left(\sqrt{\hat{\gamma}(z, u(t))}\sqrt{\gamma(z, u(t), f(t))}\right) dz \quad (22)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the observer state vector, $\hat{f}(t) \in \mathbb{R}^r$ is an estimate of actuator fault $f(t)$. Here, $q(z) \in \mathbb{R}^{p\times 1}$ can be regarded as a pre-specified weighting vector defined on $[a,\ b]$. The residual signal $\varepsilon(t)$ is formulated as an integral of difference between the estimated PDF and the measured PDF, $\hat{V}(t)$ is the observer dynamic weight vector. Further, $L \in \mathbb{R}^{n\times p}$ is the gain to be determined.

Denote

$$e_x(t) = \hat{x}(t) - x(t), \quad e_V(t) = \hat{V}(t) - V(t),$$
$$e_f(t) = \hat{f}(t) - f(t), \quad (23)$$

then the error dynamic can be obtained

$$\begin{aligned}\dot{e}_x(t) &= (A - L\Gamma_1)\, e_x(t) + G\,(e_x(t)) \\ &\quad - L\Gamma_2\,[h\,(C\hat{x}(t)) - h\,(Cx(t))] + Ee_f(t)\end{aligned} \tag{24}$$

$$e_V(t) = Ce_x(t) \tag{25}$$

where

$$\begin{aligned}G\,(e_x(t)) &= g\,(\hat{x}(t)) - g\,(x(t))\,, \\ \Gamma_1 &= \int_a^b q(z)B(z)Cdz, \\ \Gamma_2 &= \int_a^b q(z)b_n(z)dz.\end{aligned} \tag{26}$$

It can be seen that

$$\sqrt{\gamma(z, u(t), f(t))} = \Gamma_1 x(t) + \Gamma_2 h(Cx(t)) \tag{27}$$

and

$$\sqrt{\hat{\gamma}(z, u(t))} = \Gamma_1 \hat{x}(t) + \Gamma_2 h(C\hat{x}(t)). \tag{28}$$

Then the residual signal $\varepsilon(t)$ can be shown to satisfy

$$\varepsilon(t) = \Gamma_1 e_x(t) + \Gamma_2\,[h(C\hat{x}(t)) - h(Cx(t))]\,. \tag{29}$$

Since it has been assumed that the pair $(A_k,\ \Gamma_1)$ is observable, the observer gain matrix L must be determined such that $(A_k - L\Gamma_1)$ is a stable matrix.

In general, $f(t)$ represents the loss of actuator effectiveness and it is only considered based on the conventional algorithm, that is, $\dot{f}(t) = 0$. The derivative of $e_f(t)$ with respect to time can be written as

$$\dot{e}_f(t) = \dot{\hat{f}}(t). \tag{30}$$

The following theorem was obtained in [Guo and Wang, 2005].

Theorem 1. *Under Assumption 1, given scalars* $\lambda_i > 0\ (i = 1, 2)$*, if there exist matrices* $P > 0,\ R$ *with appropriate dimension,* $F \in \mathbb{R}^{r\times p}$*, an observer gain* $L \in \mathbb{R}^{n\times p}$ *and constants* $\kappa > 0,\ \theta_i\ (i = 1, 2, 3)$ *such that the following conditions hold*

$$\begin{bmatrix} \Pi_0 + \kappa I & P - \Gamma_1^T F^T \Gamma^T & \Pi_2 & 0 & C^T L_h^T \\ P - \Gamma F \Gamma_1 & 0 & 0 & \Pi_3 & 0 \\ \Pi_2^T & 0 & -I & 0 & 0 \\ 0 & \Pi_3^T & 0 & -I & 0 \\ L_h C & 0 & 0 & 0 & \theta_3 I \end{bmatrix} < 0, \tag{31}$$

and

$$L = P^{-1}R, PE_k = CF, \tag{32}$$

where $\Pi_0 = (PA - R\Gamma_1) + (PA - R\Gamma_1)^T + \frac{1}{\lambda_1} C^T L_h^T L_h C + \frac{1}{\lambda_2} L_g^T L_g$, $\Pi_2 = \left[\lambda_1 R\Gamma_2\ \lambda_2 PG\ \theta_1 R\right]$ *and* $\Pi_3 = \left[\theta_2 \Gamma F\ \theta_3 \Gamma F \Gamma_2\right]$*, then the adaptive fault estimation algorithm*

$$\dot{\hat{f}}(t) = -\Gamma F \varepsilon(t) \tag{33}$$

can realize

$$\lim_{t\to\infty} e_x(t) = 0 \ \ \textit{and} \ \ \lim_{t\to\infty} e_f(t) = 0,$$

where the symmetric positive definite matrix $\Gamma \in \mathbb{R}^{r\times r}$ *is the learning rate.*

Remark 1. *Theorem 1 gives a fault diagnosis algorithm via LMI formulation. Note that, by tuning the parameter* $\theta_i,\ (i = 1, 2, 3)$ *and* κ*, as well as* Γ*, the diagnostic error* $e_f(t)$ *can be guaranteed within a satisfactory range. From (33), actuator fault estimate using above method can be expressed as*

$$\hat{f}(t) = -\Gamma F \int_{t_f}^{t} \varepsilon(\tau)d\tau. \tag{34}$$

In fact, the conventional adaptive fault diagnosis observer using the output PDF is only integral term despite it can be guaranteed that the estimate of constant fault is unbiased. But this method is limited to systems with time-varying fault. The main limitations in use of the conventional approach are twofold. Firstly, when a larger learning rate is chosen, rapid fault estimation can be achieved, but bigger overshoot is unavoidable. Secondly, when a small learning rate is selected, the overshoot can be overcome at the cost of slow responses. The existing shortcomings are the main aims for us to develop suitable technique how to improve the performance of the fault estimation. As a matter of fact, our main contribution in this paper is to further extend the result presented in Theorem 1.

3 Adaptive Fault Diagnosis Observer for Time-Varying Faults

First, let us recall the following well-known inequality to be used later on.

Lemma 1. *Given scalar* $\mu > 0$ *and a symmetric positive definite matrix* P*, the following inequality holds*

$$2X^T PY \le \frac{1}{\mu} X^T P^2 X + \mu Y^T Y. \tag{35}$$

As for time-varying faults, due to $\dot{f}(t) \neq 0$, the derivative of $e_f(t)$ defined in (23) with respect to time is

$$\dot{e}_f(t) = \dot{\hat{f}}(t) - \dot{f}(t). \tag{36}$$

Now we are ready to present the main result of this paper. Namely, a novel adaptive fault diagnosis observer is proposed to improve the performance of the time-varying fault estimation. This main result is formulated as the following theorem which gives a modified version of the adaptive fault diagnosis observer using output probability density function (PDF).

Theorem 2. *Suppose that conditions (31) and (32) of Theorem 1 are satisfied. If for scalars $\sigma, \mu > 0$, there exist symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ and matrix $F \in \mathbb{R}^{r\times p}$ satisfying*

$$\begin{bmatrix} \Pi & -PL\Gamma_2 & E_k^T P - \frac{1}{\sigma}E_k^T P(A_k - L\Gamma_1) - F\Gamma_1 \\ * & -\mu I & \frac{1}{\sigma}E_k^T PL\Gamma_2 - F\Gamma_2 \\ * & * & \frac{1}{\sigma\mu}I - \frac{2}{\sigma}E_k^T PE_k \end{bmatrix} < 0 \tag{37}$$

where $\Pi = (A_k - L\Gamma_1)^T P + P(A_k - L\Gamma_1) + \frac{1}{\mu} + \mu L_g^2 P^2 + \frac{\mu}{\sigma}L_g^2 E_k^T PP^T E_k + \mu L_h^2 C^T C$ *and $*$ denotes the symmetric elements in a symmetric matrix. Then after a fault occurs the adaptive diagnosis algorithm*

$$\dot{\hat{f}}(t) = -\Gamma F(\dot{e}_V(t) - \sigma\varepsilon(t)) \tag{38}$$

guarantees that variables $e_x(t)$ and $e_f(t)$) are uniformly bounded.

Proof 1. *Choose the following Lyapunov function*

$$V_e(t) = e_x^T(t)Pe_x(t) + \frac{1}{\sigma}e_f(t)^T\Gamma^-1e_f(t). \tag{39}$$

From (23) and (38) the derivative with respect to time of $V_e(t)$ is

$$\begin{aligned}\dot{V}_e(t) &= e_x^T(t)\big[P(A_k - L\Gamma_1) + (A_k - L\Gamma_1)^T P\big]e_x(t) \\ &\quad + 2e_x^T(t)PG\left(e_x(t)\right) - 2e_x^T(t)PL\Gamma_2 h(Ce_x(t)) \\ &\quad + 2e_f^T(t)E_k^T E_k^T Pe_x(t) - \frac{2}{\sigma}e_f^T(t)F(\dot{e}_V(t) \\ &\quad + \sigma\varepsilon(t)) - \frac{2}{\sigma}e_f^T(t)\Gamma^{-1}\dot{f}(t).\end{aligned} \tag{40}$$

Using (32) we obtain

$$\begin{aligned}&-\frac{2}{\sigma}e_f^T(t)F(\dot{e}_V(t) + \sigma\varepsilon(t)) \\ &= -\frac{2}{\sigma}e_f^T(t)E_k^T P\dot{e}_x(t) - 2e_f^T(t)F\varepsilon(t).\end{aligned} \tag{41}$$

Substituting (29) and (41) into (40) yields

$$\begin{aligned}\dot{V}_e(t) &= e_x^T(t)\big[P(A_k - L\Gamma_1) + (A_k - L\Gamma_1)^T P\big]e_x(t) \\ &\quad + 2e_x^T(t)PG\left(e_x(t)\right) - 2e_x^T(t)PL\Gamma_2 h(Ce_x(t)) \\ &\quad + 2e_f^T(t)E_k^T Pe_x(t) - \frac{2}{\sigma}e_f^T(t)E_k^T P\big[(A_k - L\Gamma_1) \\ &\quad e_x(t) + G(e_x(t)) - L\Gamma_2 h(E_k e_x(t)) + E_k e_f(t)\big] \\ &\quad - 2e_f^T(t)F\big[\Gamma_1 e_x(t) + \Gamma_2 h(E_k e_x(t)\big] - \frac{2}{\sigma}e_f^T(t) \\ &\quad \Gamma^{-1}\dot{f}(t).\end{aligned} \tag{42}$$

As the nonlinear term $h(Ce_x(t))$ satisfies the Lipschitz condition, so for a scalar μ we have

$$\mu L_h^2 e_x^T(t)C^T Ce_x(t) - \mu h^T\left(Ce_x(t)\right)h\left(Ce_x(t)\right) \geq 0. \tag{43}$$

From Lemma 1, it is easy to show that

$$\begin{aligned}&2e_x^T(t)PG\left(e_x(t)\right) \leq \frac{1}{\mu}e_x^T(t)e_x(t) + \mu L_g^2 e_x^T(t)P^2 e_x(t), \\ &\frac{2}{\sigma}e_f^T(t)\Gamma^{-1}\dot{f}(t) \leq \frac{1}{\mu\sigma}e_f^T(t)e_f^T(t) + \frac{\mu}{\sigma}\dot{f}(t)\Gamma^{-1}\Gamma^{-1}\dot{f}(t) \\ &\leq \frac{1}{\mu\sigma}e_f^T(t)e_f^T(t) + \frac{\mu}{\sigma}f_1^2\lambda_{max}(\Gamma^{-1}\Gamma^{-1}).\end{aligned} \tag{44}$$

Therefore substituting (44) into (42) and calculating the derivative of $V_e(t)$ yields

$$\begin{aligned}\dot{V}_e(t) =& e_x^T(t)\big[P(A_k - L\Gamma_1) + (A_k - L\Gamma_1)^T P\big]e_x(t) \\ &+ \frac{1}{\mu}e_x^T(t)e_x(t) + \mu L_g^2 e_x^T(t)P^2 e_x(t) \\ &- 2e_x^T(t)PL\Gamma_2 h(Ce_x(t)) + 2e_f^T(t)E^T Pe_x(t) \\ &- \frac{2}{\sigma}e_f^T(t)E_k^T P(A_k - L\Gamma_1)e_x(t) \\ &+ \frac{1}{\mu\sigma}e_f^T(t)e_f(t) \\ &+ \frac{\mu}{\sigma}L_g^2 e_x^T(t)E_k^T PP^T E_k e_x(t) \\ &+ \frac{2}{\sigma}e_f^T(t)E_k^T PL\Gamma_2 h(Ce_x(t)) \\ &- \frac{2}{\sigma}e_f^T(t)E_k^T PE_k e_f(t) \\ &- 2e_f^T(t)F\big[\Gamma_1 e_x(t) + \Gamma_2 h(E_k e_x(t))\big] \\ &+ \frac{1}{\mu\sigma}e_f^T(t)e_f^T(t) + \frac{\mu}{\sigma}f_1^2\lambda_{max}(\Gamma^{-1}\Gamma^{-1}) \\ &+ \mu L_h^2 e_x^T(t)C^T Ce_x(t) \\ &- \mu h^T\left(Ce_x(t)\right)h\left(Ce_x(t)\right) \\ &= \Phi(t)^T\Omega\Phi(t) + \delta\end{aligned} \tag{45}$$

where

$$\Omega = \begin{bmatrix} \Pi & -PL\Gamma_2 & E_k^T P - \frac{1}{\sigma}E_k^T P(A_k - L\Gamma_1) - F\Gamma_1 \\ * & -\mu I & \frac{1}{\sigma}E_k^T PL\Gamma_2 - F\Gamma_2 \\ * & * & \frac{1}{\sigma\mu}I - \frac{2}{\sigma}E_k^T PE_k \end{bmatrix},$$

$$\Pi = (A_k - L\Gamma_1)^T P + P(A_k - L\Gamma_1) + \frac{1}{\mu} + \mu L_g^2 P^2 + \frac{\mu}{\sigma}L_g^2 E_k^T PP^T E_k + \mu L_h^2 C^T C,$$

$$\Phi(t) = \begin{bmatrix} e_x(t) \\ h(Ce_x(t)) \\ e_f(t) \end{bmatrix}, \ \delta = \frac{\mu}{\sigma}f_1^2 \lambda_{max}(\Gamma^{-1}\Gamma^{-1}).$$

Under persistently exciting input $u(t)$, if the matrix $\Omega < 0$, we can obtain that $\dot{V}_e(t) < -\lambda_{min}(\Omega)\|\Phi(t)\|^2 + \delta$. It follows that $\dot{V}_e(t) < 0$ for $\lambda_{min}(\Omega) > \delta$, which can guarantee asymptotic convergence of estimation errors of both the state and fault. Therefore, variables $(e_x(t),\ e_f(t))$ are uniformly bounded. This is the end of proof.

Remark 2. *It it easy to shown that the inequality (37) in Theorem 2 can be solved by LMI toolbox. But the solving difficulty is added because of the equation $L = P^{-1}R$. More specifically, it is problem how to solve (32) and (37) simultaneously. This open problem needs to be addressed in the future.*

4 Simulation Result and Analysis

In this section we consider the following spacecraft attitude control system with the corresponding parameters to show the effectiveness of the proposed method. The main inertia, J of the spacecraft is given as follows

$$J = \begin{bmatrix} 12.49 & 0 & 0 \\ 0 & 13.85 & 0 \\ 0 & 0 & 15.75 \end{bmatrix} kg.m^2 \quad (46)$$

and the orbital ω_0 is $7.223 \times 10^-5[rad/s]$. The state vector is chosen as $x = [\phi\ \theta\ \psi\ \omega_1\ \omega_2\ \omega_3]^T$, therefore matrices A_k, B_k and continuous nonlinear function $g_k(x(t), t)$ can be obtained. For spacecraft attitude control system, the six variable are available, so C is identity matrix.

In this particular situation, an actuator fault will occur in the input channel and the actuator fault distribution matrix $E_k = B_k$. The square root output PDF can be approximated by seven base functions, that is, for $i = 1, 2, \ldots, 7$,

$$b_i(z) = \exp(-(z - \mu_i)^2 \sigma_i^{-2}), \quad (47)$$

where $z \in [0,\ 0.5]$, $\mu_i = 0.003 + 0.006(i - 1), \sigma_i = 0.003$. Selecting, $q(z) = 1$, it is easy to compute that

$$\Gamma_1 = \begin{bmatrix}0.0063\ 0.0075\ 0.0075\ 0.0075\ 0.0075\ 0.0075\end{bmatrix},$$
$$\Gamma_2 = \begin{bmatrix}0.0075.\end{bmatrix}$$

Solving (32) and (37) using Matlab LMI Toolbox, one can obtain that

$$\eta = 1.5001,\ P = \begin{bmatrix} 1.91 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 1.91 & 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0.91 & 0 & 0 & 0.2 \\ 0.2 & 0 & 0 & 1.95 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 1.95 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 1.95 \end{bmatrix},$$

$$R = \begin{bmatrix} 0.0102 \\ 0.0153 \\ 1.7344 \\ -0.0070 \\ -0.0003 \\ 0.3682 \end{bmatrix},\ F = \begin{bmatrix} 0.016 & 0 & 0 \\ 0 & 0.0144 & 0 \\ 0 & 0 & 0.0127 \\ 0.1561 & 0 & 0 \\ 0 & 0.1408 & 0 \\ 0 & 0 & 0.1238 \end{bmatrix},$$

$L = [0.0058\ 0.0081\ \ 1.9074\ \ -0.0042\ \ -0.0010\ \ -0.0068]^T$.

Taking the learning rate $\Gamma = diag(10, 10, 10, 10, 10, 10)$ and sampling time $T = 0.01$s, the system is subject to the reference input $u(t) = [1\ 1\ 1]^T$ and the initial value $x(0) = [0,\ 0,\ 0,\ 0,\ 0,\ 0]^T$. In order to show that proposed method is superior to the conventional one, we will compare them with the following simulations. There are two cases for actuator fault $f(t) = [f_1(t)\ \ f_2(t)]^T$.

Assume that constant actuator fault is created as

$$f_1(t) = \begin{cases} 0, & 0 \le t \le 3 \\ 0.6, & 3 < t \le 10 \\ 0.3, & 10 < t \le 20, \end{cases} \quad f_2(t) = 0. \quad (48)$$

The simulation results for constant fault estimation using conventional and the proposed method are shown in Fig. 2 and Fig. 3.

Then the time-varying actuator fault is considered as

$$f_2(t) = \begin{cases} 0, & 0 \le t < 3 \\ 0.2sin(5t - 10), & 3 < t \le 20 \end{cases} \ f_1(t) = 0. \quad (49)$$

Fig. 4 and Fig. 5 demonstrate the simulation results for time-varying fault estimation using the above two mentioned methods.

Simulation results for faults estimation are illustrated in Figs. 2, 3, 4 and 5. From these simulations, it can be seen that for constant fault, asymptotic convergence of fault estimation error can be both achieved

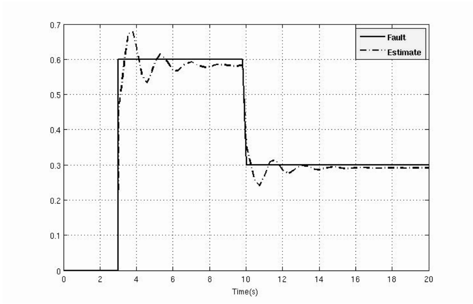

Figure 2: Constant fault $f(t)$ and its estimate $\hat{f}(t)$ using the conventional

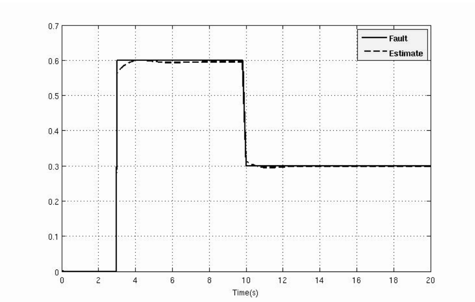

Figure 3: Constant fault $f(t)$ and its estimate $\hat{f}(t)$ using the proposed method.

using the two methods. But the proposed method can improve the performance of the fault estimation at the faster speed. As for time-varying fault, the proposed method can also achieve more satisfactory performance of the time-varying estimation than the conventional one. Compared with the conventional method for both constant and time-varying fault, we can conclude that the proposed approach provides much better performance.

5 Conclusion and Outlooks

In this paper, a novel fault diagnosis scheme using output probability density estimation is proposed to improve the performance of the fault estimation. The proposed method improves the speed of the fault estimation and eliminate steady estimation error simultaneously. The application of this design scheme to a satellite attitude control system shows that the systemic fault can be detected and estimated with satisfactory performance. Unlike classical FDI problem, the measured output of the system is viewed as a stochastic process and its probability density function (PDF) is modeled with B-spline functions, which leads to a deterministic space-time dynamic model including nonlinearities and uncertainties. For this model, a new fault diagnosis approach has been presented using LMI formulation. From simulation results, it can be seen that, the proposed method can improve the performance of the fault estimation, including constant and

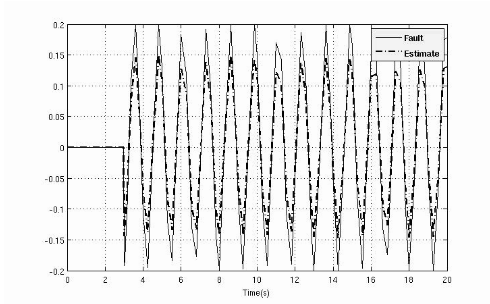

Figure 4: Time-varying fault $f(t)$ and its estimate $\hat{f}(t)$ using the conventional

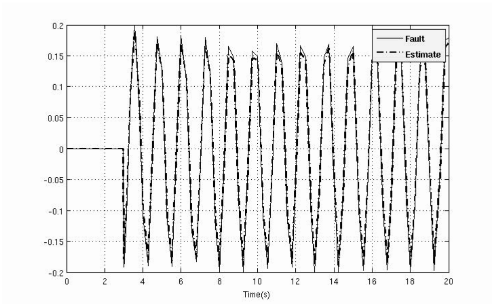

Figure 5: Time-varying fault $f(t)$ and its estimate $\hat{f}(t)$ using the proposed method.

time-varying fault. Simulation examples are given to demonstrate the efficiency of the proposed approach.

Finally, there are still many open problems to be further investigated. The first problem concerns conditions about the solvability of LMI formulation for the proposed method. The second problem concerns extension of the proposed algorithm to slow-drifting faults. These problems are very difficult and they need to be concerned in the future.

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