DECENTRALIZED MODEL REFERENCE ADAPTIVE PRECIZE CONTROL OF COMPLEX OBJECTS

S.D. Zemlyakov , V.Yu. Rutkovsky, V.M. Glumov and V.M. Sukhanov

Institute of Control Science by V.A. Trapeznikov Russian Academy of Science
65 Profsoyuznaya , 117997, Moscow, Russia
Tel: 007 (950)3348730; e-mail: zeml@ipu.ru

Abstract: This paper deals with the control problem of complex objects on the base of the model reference adaptive principle. For decomposition and precise control new algorithms are discovered. A space robotic module is considered as the example of a complex object. Computer simulation demonstrates good results.

Key words: Complex Object, Decomposition, Model Reference Adaptive Control, Lyapunov Function, Simulation

1. INTRODUCTION

As a complex we consider an object with some interconnected subsystems (Voronov, 1985; Šiljak, 1991). A mathematical model (MM) of such an object is usually nonlinear and nonstationary one. Synthesis of control algorithms for such an object is not a simple problem. Usual if not a single method for this goal is decomposition and aggregation (Voronov, 1985; Šiljak, 1991). The decomposition could be realized on "physical" or "mathematical" principles (Šiljak, 1991). For every subsystem a local good control (in any sense) is discovered provided that interconnections are missing. Then it is necessary to prove that the good behavior of the system on the whole takes place (Voronov, 1965; Šiljak, 1991).

In this paper we use other an approach: for every subsystem a component of interconnections is selected and compensated on the base of adaptive control. More correctly the approach includes the use

- of computer aided control (Zemlyakov and Rutkovsky, 2004);
- of a programmed adaptive control;
- of a model reference adaptive control (Zemlyakov and Rutkovsky, 1966; Pertrov, *et al*., 1980).

A local adaptive control system for an object subsystem could be synthesized on the base of traditional algorithms for a model reference adaptive approach. In result the adaptive system will be nonlinear and nonstationary. It is known that for such a system is difficult to supply good dynamics not speaking about precise control (Zemlyakov and Rutkovsky, 1966; Pertrov, *et al*., 1980). So in this paper we derive new nontraditional algorithms for a system with reference model that are capable to guarantee the desiered precise control.

2. PROBLEM STATEMENT

Consider a complex object (Zemlyakov and Rutkovsky, 2004) described by the differential equation

$$A(q)\ddot{q} + \sum_{s=1}^{n}\left[\dot{q}^T D_s(q)\dot{q}\right]e_s = S(q)M \tag{1}$$

where $q \in R^n$, $M \in R^n$, $A(q) = (a_{ij}(q)) > 0$, $D_s(q) = (d_{it}^s(q))$,

$$d_{it}^s = \frac{1}{2}\left[\frac{\partial a_{si}(q)}{\partial q_t} + \frac{\partial a_{st}(q)}{\partial q_i} - \frac{\partial a_{it}(q)}{\partial q_s}\right]$$
$$(i, t, s = \overline{1, n}).$$

During the object's operating

- matrices $A(q)$, $D_s(q)$, $S(q)$ $(s=\overline{1, n})$ are known (Zemlyakov and Rutkovsky, 2004);
- vectors $q = q(t)$, $\dot{q} = \dot{q}(t)$ are measurable.

For every q_i $(i = \overline{1, n})$ there exists a function $q_i^0(t)$ and an equation

$$\ddot{q}_i + d_i\dot{q}_i + k_i q = k_i q_i^0(t)\,, \tag{2}$$

where the function $q_i^0(t)$ and the numbers $k_i > 0,\ d_i > 0$ are prescribed in advance.

The problem:

It is necessary to discover control algorithms

$$M = M(t, q, \dot{q})$$

that guarantees the motion (2).

3. DECOMPOSITION OF AN OBJECT MATHEMATICAL MODEL

Let an object with MM (1) is decomposed to N subsystems according to the physical principle (Šiljak, 1991)

$$\sum_{j=1}^{N} A^{ij}(q)\ddot{q}^j + N^i(q,\dot{q}) = \sum_{j=1}^{N} S^{ij}(q)M^j$$
$$(i = \overline{1,\ N}),$$

where $A^{ii}(q) > 0,\quad q^j,\ M^j$ are components of vectors q, M

$$q^T = (q^1,\ q^2,\ \ldots,\ q^N),$$
$$M^T = (M^1,\ M^2,\ \ldots,\ M^N).$$

Dimensions of the vectors $q^j,\ M^j\ \ (j = \overline{1,\ N})$ are equal n^j and

$$\sum_{j=1}^{N} n^j = n\,.$$

The MM for any subsystem could be written in the form

$$A^{ii}(q)\ddot{q}^i = S^{ii}(q)M^i + F^i(t,q,\dot{q},\ddot{q})$$
$$(i = \overline{1,\ N}), \tag{3}$$

where $A_{ii}(q) \in R^{n^i \times n^i}$, $A_{ii}(q) > 0,\ \ M^i \in R^{n^i}$,

$$F^i(*) = \sum_{\substack{j=1 \\ j\neq i}}^{N} S^{ij}(q)M^j -$$
$$-\sum_{\substack{j=1 \\ j\neq i}}^{N} A^{ij}(q)\ddot{q}^j - N^i(q,\dot{q})\,.$$

4. MATHEMATICAL MODEL OF A SUBSYSTEM WITH ACTUATORS

Different subsystems could have actuators of different nature. In this paper we consider only dc motors as actuators. The MM of dc motor is well known (Krutiko, 1991)

$$J_j^i \ddot{q}_j^i = \frac{1}{r_j^i} M_j^{id} - \frac{1}{(r_j^i)^2} M_j^i,$$
$$\tau_j^i \dot{M}_j^{id} + M_j^{id} = \frac{k_{mj}^i}{R_i} u_j^i - \frac{k_{mj}^i k_{\omega j}^i}{R_i} r_j^i \dot{q}_j^i \tag{4}$$
$$(i = \overline{1,\ N^l};\quad j = \overline{1,\ n^i}),$$

where N^l is the number of subsystems with dc motor actuators.

In (4) M_j^{id} is a moving moment of a motor; M_j^i is a moment for a load rotation; r_j^i is a reduction coefficient; $J_j^i,\ R_j^i,\ \tau_j^i, k_{mj}^i, k_{\omega j}^i$ are motor constructive parameters; u_j^i are control algorithms to be discovered.

Let the MM of an object subsystem with dc motor actuators has the form (3). To the equation (3) it is necessary to add the equations for actuators (4) presented in matrix form

$$A^{il}\ddot{q}^i = R^i(M^{id} - M^{il}),$$
$$\tau^i \dot{M}^{id} + M^{id} = \rho^i U^{il} - \beta^i \dot{q}^i \tag{5}$$
$$(i = \overline{1,\ N^l}),$$

where

$$A^{il} = diag(J_1^i (r_1^i)^2,\ J_2^i (r_2^i)^2, \ldots, J_{n^i}^i (r_{n^i}^i)^2),$$
$$R^i = diag(r_1^i,\ r_2^i, \ldots,\ r_{n^i}^i),$$
$$(M^{id})^T = (M_1^{id},\ M_2^{id}, \ldots, M_{n^i}^{id}),$$
$$(M^{il})^T = (M_1^{il},\ M_2^{il}, \ldots, M_{n^i}^{il}),$$
$$\tau^i = diag(\tau_1^i,\ \tau_2^i, \ldots, \tau_{n^i}^i),$$
$$\rho^i = diag(\frac{k_{m1}^i}{R_1^i},\ \frac{k_{m2}^i}{R_2^i}, \ldots, \frac{k_{mn^i}^i}{R_{n^i}^i}),$$
$$\beta^i = diag(\frac{k_{m1}^i k_{\omega 1}^i}{R_1^i},\ \frac{k_{m2}^i k_{\omega 2}^i}{R_2^i}, \ldots, \frac{k_{mn^i}^i k_{\omega n^i}^i}{R_{n^i}^i})\,.$$

From (3) and (5) the MM of a subsystem movement together with actuators accepts the form

$$
\begin{gathered}
[A^{ii}(q)+S^{ii}(q)A^{il}]\ddot{q}^i = \\
= S^{ii}(q)RM^{id} + F^i(t,q,\dot{q},\ddot{q}), \\
\tau^i \dot{M}^{id} + M^{id} = \rho U^{il} - \beta^i \dot{q}^i, \\
(i=\overline{1,\ N^l}).
\end{gathered} \tag{6}
$$

For control algorithms synthesis it is possible (Krutiko, 1991) to take the condition

$$\tau^i = 0$$

and to simplify the system (6) to the form

$$
\begin{gathered}
[A^{ii}(q)+S^{ii}(q)A^{il}]\ddot{q}^i = S^{ii}(q)R^i\rho^i U^{il} - \\
- S^{ii}(q)R^i\beta^i\dot{q}^i + F^i(t,q,\dot{q},\ddot{q}).
\end{gathered} \tag{7}
$$

5. ADAPTIVE PROGRAMMED CONTROL OF AN OBJECT SUBSYSTEM

The MM of an object subsystem with dc motor actuators (7) could be presented in the form

$$
\begin{gathered}
\ddot{q}^i = [A^{ii}(q)+S^{ii}(q)A^{il}]^{-1}S^{ii}(q)R^i\rho^i U^{il} + \\
+ f^i(t,q,\dot{q},\ddot{q}),
\end{gathered} \tag{8}
$$

where

$$
\begin{gathered}
f^i(t,q,\dot{q},\ddot{q}) = [A^{ii}(q)+S^{ii}(q)A^{il}]^{-1}\times \\
\times[F^i(t,q,\dot{q},\ddot{q}) - S^{ii}(q)R^i\beta^i\dot{q}^i].
\end{gathered}
$$

The condition

$$\det(A^{ii}(q)+S^{ii}(q)A^{il}) \neq 0$$

is usually right.

Let us take a control algorithm in the form

$$
\begin{gathered}
U^{il} = (S^{ii}(q)R^i\rho^i)^{-1}[A^{ii}(q)+S^{ii}(q)A^{il}]\times \\
\times[K^i(q^{i0}-q^i) - D^i\dot{q}^i + S^i],
\end{gathered} \tag{9}
$$

where $(q^{i0})^T = (q_1^{i0}(t),\ q_2^{i0}(t),\dots,q_{n^i}^{i0}(t))$,
$K^i = diag(k_1^i,\ k_2^i,\dots,k_{n^i}^i)$,
$D^i = diag(d_1^i,\ d_2^i,\dots,d_{n^i}^i)$,
$(S^i)^T = (S_1^i,\ S_2^i,\dots,S_{n^i}^i)$.

Equalities (8) and (9) together give an equation

$$
\begin{gathered}
\ddot{q}^i + D^i\dot{q}^i + K^iq^i = K^iq^{i0}(t) + \\
+ [f^i(t,q,\dot{q},\ddot{q}) + S^i].
\end{gathered} \tag{10}
$$

From the equation (10) we see that if it is valid the equality

$$[f^i(t,q,\dot{q},\ddot{q}) + S^i] \equiv 0 \tag{11}$$

then the movement of a subsystem with number $i \in N^l$ is decomposed on n^i separate subsubsystems with MM

$$
\begin{gathered}
\ddot{q}_j^i + d_j^i\dot{q}_j^i + k_j^iq_j^i = k_j^iq_j^{i0}(t) \\
\left(j=\overline{1,\ n^i}\right).
\end{gathered} \tag{12}
$$

From (2) and (12) we see that the control algorithm (9) gives the problem solution if the equality (11) takes place.

6. MODEL REFERENCE ADAPTIVE CONTROL

Below we will try to hold up the equality (11) on the base of model reference adaptive control (Pertrov, *et al.*, 1980). Adaptive algorithms will not be traditional as in (Zemlyakov and Rutkovsky, 1966; Pertrov, *et al.*, 1980) but more constructive for precise control.

If the equality (11) does not take place than the equation (12) takes the form

$$
\begin{gathered}
\ddot{q}_j^i + d_j^i\dot{q}_j^i + k_j^iq_j^i = k_j^iq_j^{i0}(t) + f_j^i(t) + S_j^i \\
\left(j=\overline{1,\ n^i}\right),
\end{gathered} \tag{13}
$$

where $f_j^i(t) = f_j^i(t,q,\dot{q},\ddot{q})$.

Let us choose a reference model in the form

$$
\begin{gathered}
\ddot{q}_{mj}^i + d_j^i\dot{q}_{mj}^i + k_j^iq_{mj}^i = k_j^iq_j^{i0}(t) \\
\left(j=\overline{1,\ n^i}\right)
\end{gathered} \tag{14}
$$

and with the equality

$$\varepsilon_j^i = q_j^i - q_{mj}^i$$

we receive from (13) and (14) the equation with respect to the error ε_j^i in the form

$$
\begin{gathered}
\ddot{\varepsilon}_j^i + d_j^i\dot{\varepsilon}_j^i + k_j^i\varepsilon_j^i = [f_j^i(t) + S_j^i] \\
\left(j=\overline{1,\ n^i}\right).
\end{gathered} \tag{15}
$$

With notation

$$\varepsilon_j^i = x_{j1}^i,$$
$$\dot{\varepsilon}_j^i = x_{j2}^i,\ f_j^i(t) + S_j^i = y_j^i,$$
$$\dot{f}_j^i(t) = \mu_j^i(t),$$
$$\dot{S}_j^i = \psi_j^i\ ,$$

equation (15) could be written in the matrix form

$$\begin{aligned} \dot{x}_j^i &= A_j^i x_j^i + \beta_j^i, \\ \dot{y}_j^i &= \psi_j^i + \mu_j^i(t), \end{aligned} \tag{16}$$

where

$$(x_j^i)^T = (x_{j1}^i\,,\ x_{j2}^i),\ (\beta_j^i)^T = (0\ \ y_j^i)\,,$$
$$A_j^i = \begin{pmatrix} 0 & 1 \\ -k_j^i & -d_j^i \end{pmatrix}.$$

We will try to choose an algorithm for S_j^i purposeful variation from the condition of the asymptotical convergence of the system (16) movement with respect to the movement

$$x_j^i \equiv 0,\quad y_j^i \equiv 0.$$

For this goal we take a Lyapunov function in the form

$$V_j^i(x_j^i, y_j^i) = \kappa_j^i (x_j^i)^T P_j^i (x_j^i) + (y_j^i)^2 \tag{17}$$

where P_j^i is a positive definite matrix, $\kappa_j^i = const > 0$. The derivative of $V_j^i(x_j^i, y_j^i)$ with respect to time along a solution of the system (16) is the equality

$$\begin{aligned} \dot{V}_j^i(x_j^i, y_j^i) &= \kappa_j^i (x_j^i)^T Q_j^i (x_j^i) + \\ &+ 2y_j^i[\sigma_j^i + \mu_j^i(t) + \psi_j^i], \end{aligned} \tag{18}$$

where Q_j^i is the prescribed negative definite matrix, $\sigma_j^i = (p_{21}^i x_{j1}^i + p_{22}^i x_{j2}^i)$, p_{rk}^i are elements of the matrix $P_j^i = (p_{rk}^i)\ \ (r,k = 1,2)$.

For an analytical result we suppose that the sign of the coordinate y_j^i is known. Really it is possible only to approach to this assumption by different way. In the paper we do not concentrate on this question. One simple possible way will be accepted for simulation. Then we choose the desired algorithm in the form

$$\psi_j^i = -\sigma_j^i - \overline{k_j^i} sign(y_j^i)\,, \tag{19}$$

where $\overline{k_j^i} > 0$.

We suppose that the inequality

$$\overline{k_j^i} > \left|\mu_j^i(t)\right|$$

takes place. Then we have the result

$$V_j^i(x_j^i, y_j^i) > 0, \qquad \dot{V}_j^i(x_j^i, y_j^i) < 0$$

which ensure the solution of the problem.

7. SIMULATION RESULTS

For simulation we consider a space robotic module (SRM) (Zemlyakov and Rutkovsky, 2004) as a complex object with MM (1). As a mechanical object SRM has a big number of degrees of freedom. MM of an SRM is nonlinear multiconnected nonstationary one. Automatic control by such an object is not a simple problem. At the same time SRM operating demands precise dynamic accuracy to provide for its safety, for the safety of objects with which it interacts.

On the base of "physical principle" (Šiljak, 1991) the MM of SRM could be divided on two interconnected subsystems: subsystem for the carrying body and subsystem for manipulators (Zemlyakov and Rutkovsky, 2004). Control algorithms for the carrying body subsystem could be synthesized on the base of rely control, Pontryagin maximum principle and the direct Lyapunov method (Zemlyakov and Rutkovsky, 2004). Here we will try to synthesize control algorithms for the manipulators subsystem on the base of adaptive control (Zemlyakov and Rutkovsky, 1966; Pertrov, *et al.*, 1980).

We assume that the decomposition problem is solved and now it is necessary to show that algorithms (19) really provide precise control of $q_i(t)$ with respect to $q_i^0(t)$ in (2).

Let $k_j^i = 0.25$ and $d_j^i = 0.7$ in (13), $Q_j^i = \begin{pmatrix} -0.05 & 0 \\ 0 & -0.05 \end{pmatrix}$ in (18). Then $P_j^i = \begin{pmatrix} 0.11 & 0.10 \\ 0.10 & 0.18 \end{pmatrix}$ in (17) and hence $\sigma_j^i = (0.1x_{j1}^i + 0.18x_{j2}^i)$ in (18).

Let $q_j^{i0}(t)=\sin(0.15t)$. In fig.1.a under the number 1 we see $k_j^i q_j^{i0}(t)$ and under the number 2 $k_j^i q_j^{i0}(t)+f_j^i(t)$. The term $f_j^i(t)$ disturbs $k_j^i q_j^{i0}(t)$ significantly. In fig.1.b under numbers 1 and 2 respectively we see $q_j^i(t)$ and $q_{mj}^i(t)$ under condition $S_j^i(t)\equiv 0$. Of course without adaptation in fig.1.b the difference $\varepsilon_j^i(t)=q_j^i(t)-q_{mj}^i(t)$ is significant. In fig.1.c $S_j^i(t)\neq 0$. We see that $q_j^i(t)$ practically coincides with $q_{mj}^i(t)$.

Fig.2 shows the same result for another function $q_j^{i0}(t)$.

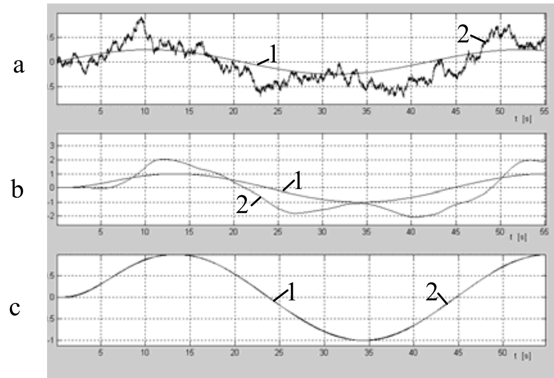

Fig. 1.

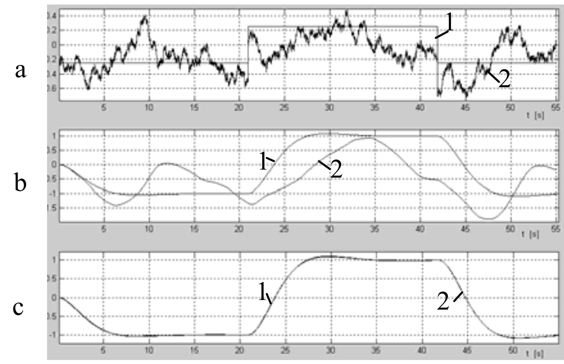

Fig. 2.

ACKNOWLEDGEMENT

The work reported in this paper is a contribution to project 05-08-18175 funded by Russian Foundation for Basic research for which authors are grateful.

REFERENCES

Voronov A.A., (1985).*Introduction to Dynamics of Complex Control Systems.* Nauka, Moscow, (Russian).

Šiljak D., (1991). *Decentralized Control of Complex Systems.* Academic Press, Inc., New York,.

Zemlyakov S.D. and Rutkovsky V.Yu. (2004). Computer – Aided Modelling and Analytical Synthesis of Control Algorithms for a Spacecraft with Discretely changing Structure. In: *Proc. of the 16-th IFAC Symposium on Aut. Control in Space.* State University of Aerospace Instrumentation. Saint-Petersburg. Russia.

Zemlyakov S.D. and Rutkovsky V.Yu. (1966). Synthesis of Model Reference Adaptive Control System. In: *Automatika and Telemechanika.* **№3**. P.70 – 77. Nauka, Moscow (Russian).

Petrov B.N., Rutkovsky V.Yu. and Zemlyakov S.D. (1980). *Adaptive Coordinate-Parametric Control of Nonstationary Objects.* Nauka, Moscow (Russian).

Krutiko P.D. (1991). *Control of Robot Actuators.* Nauka, Moscow (Russian).