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MAXIMIZING RADIOFREQUENCY HEATING ON FTU VIA EXTREMUM SEEKING: PARAMETER SELECTION AND TUNING

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Abstract

In this paper we illustrate the use of a novel extremum seeking scheme recently proposed in [2] to minimize the percentage of reflected power on the Frascati Tokamak Upgrade (FTU) experimental facility during radiofrequency heating. The paper contains an explanation of how the parameters of the extremum seeking scheme should be selected to induce desirable closed-loop performance. The effectiveness of the tuning procedure will be shown via numerical simulations.

Key words

Nonlinear control systems, Optimization, Tokamak plasmas

1 Introduction

Since the early 1950s the "extremum seeking" control has been introduced to minimize/maximize unknown functions at the output of dynamical systems (see [5] and [9]). In [8], for the first time, local stability properties of an extremum seeking feedback scheme for general nonlinear systems has been formally proved, motivating further interesting results (see [1], [6], [10]). Recently, in [11] an extremum controller slightly different from the one in [8] has been shown, under slightly stronger conditions, to formally guarantee non-local (semiglobal practical) stability properties.

An application that recently benefited from the use of extremum seeking techniques is that of control of Tokamak plasmas. Most of the heating of the current Tokamak experiments is obtained by Joule effect by way of a high current flowing in the plasma. However, since the plasma resistivity drops down as the temperature increases, alternative heating methods are necessary, especially in future experiments such as ITER [7], which is the ultimate worldwide international experiment jointly built by the international community in Cadarache (France). Among these alternative methods, radiofrequency heating seems to be the most promising and definitely the most widely experimented one. In this method, high frequency waves are delivered to the plasma via suitable antennas and the corresponding energy is absorbed by the plasma and transformed into heat via the same phenomenon that happens in microwave ovens: resonant modes of ions and electrons (or even hybrid resonances). The reason why extremum seeking is needed in these applications is that there's an important coupling between the antenna and the plasma Scrape Off Layer (SOL), namely the outer surface of the plasma within the Tokamak vacuum vessel. If the coupling is poor, then reflected waves can damage the antenna and typically cause undesired safety shutdowns. Moreover, the effectiveness of the radiofrequency heating is evidently proportional to the coupling between antenna and plasma, because optimized coupling causes maximum absorbed power, therefore temperature increase.

In [13], some experimental results on the Frascati Tokamak Upgrade (FTU) [12], an experiment owned by ENEA in Frascati (Rome, Italy), showed that when using the Lower Hybrid (LH) antennas of FTU, naive solutions to the problem of minimizing the reflected power already gave desirable performance improvements. The problem with these early solutions was slow convergence and lack of any guarantee. Later experiments employed a modified extremum seeking technique to solve the same problem [4], which showed increased performance and robustness in experiments, as compared to the previous solution of [13], even though from the experimental results it was evident that the algorithm employed had some space for improvement. Finally, implementation issues arising from the use of multiple antennas were reported in [3].

In this paper we discuss about parameters tuning of the new extremum seeking scheme proposed in [2]. The choice depending on the plant and noise properties is discussed and shown via simulation examples The paper is organized as follows. In Section 2 the control scheme is recalled and some general ideas on the parameters tunings are outlined. Section 3 illustrates by two examples the extremum seeking construction and new phenomena related to the parameter selection. Conclusions are given in 4.

2 The control scheme

In this section we recall the control scheme proposed in [2], whose aim is to find a reference signal for a dynamical system such that an unknown function of its output is minimized. The control scheme that we consider to deal with this problem is shown in Figure 1. The unknown map is $g(\cdot)$, with input y and d, the output of the first order linear dynamical system and the disturbance signal, respectively. The parameter $\varepsilon > 0$ sets the convergence speed of y to $\sigma\theta$, where $\delta > 0$ is the static gain of the linear plant. The noises ν_1 and ν_2 affect the measurements which are filtered by two SISO systems $F(s)$. The output of a unit saturation is fed with the signal $k_2 z_1(t) z_2(t)$ and is integrated and multiplied by k_1, yielding the plant reference $\theta(t)$, with positive scalars k_1 and k_2.

Note that the first order plant that we are considering can be the approximation of a higher order asymptotically stable system. The output difference between the real plant and its approximation may be enclosed in the signal $d(t)$. Assumptions about the unknown function $g(\cdot)$ and the signals $d(t)$, $\nu_1(t)$, and $\nu_2(t)$ are introduced next.

Assumption 1. *The unknown map $g(\cdot) : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz, locally bounded and there exist a $y^\star \in \mathbb{R}$ and a class $\mathcal{K}$ function $\gamma(\cdot) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that for almost all $s \in \mathbb{R}$:*

$$\nabla g(s)(s - y^\star) \geq |s - y^\star| \gamma(|s - y^\star|). \tag{1}$$

□

This assumption implies that $g(\cdot)$ is in the incremental sector $(0, \infty)$ around $y^\star$, its minimum.

Assumption 2. *The disturbance $d(\cdot)$ is bounded and has bounded first ($|\dot{d}(t)| \leq \bar{d}$) and second time derivatives, moreover, it is such that there exist $T > 0$ and $c > 0$ satisfying*

$$\int_t^{t+T} |\dot{d}(\tau)| d\tau \geq c \tag{2}$$

for all $t \geq 0$. The noise signals ν_1 and ν_2 are bounded and with bounded derivatives. □

Without any noise, $\nu_1 = \nu_2 = 0$, and under the hypothesis that the filters can compute the ideal derivative, that is $F(s) = s$, the closed loop dynamics is

$$\begin{aligned} \varepsilon \dot{y} &= -y + \delta\theta, \\ z_1(t) &= \dot{y}(t) + \dot{d}(t), \\ z_2(t) &= \frac{\partial g(y(t) + d(t))}{\partial y} \left(\dot{y}(t) + \dot{d}(t) \right), \\ \dot{\theta} &= -k_1 \mathrm{sat}\left(k_2 z_2(t) z_1(t)\right), \end{aligned} \tag{3}$$

and the following theorem stated in [2] holds.

Theorem 1. *Under Assumptions 1-2, for any positive constants k_1 and k_2, the closed-loop system (3) is such that both $y(\cdot)$ and $\theta(\cdot)$ are bounded, the set $\mathcal{A} = \mathcal{B}(y^\star, 2\varepsilon k_1 + \bar{d})$ with $\bar{d} := \|d(\cdot)\|_\infty$ is eventually forward invariant [1] and attractive and*

$$g(y(t)) \leq \max_{a \in \mathcal{A}} \{g(a), g(y(0))\}, \; \forall \, t \geq 0.$$

□

Note that the saturation block in the feedback loop limits $\dot{\theta}$ below k_1. This is an appealing property for "risky" plants or when rapidly changing signals may excite high frequency dynamics. Moreover, this approach allows to meet rate saturation constraints of the actuators.

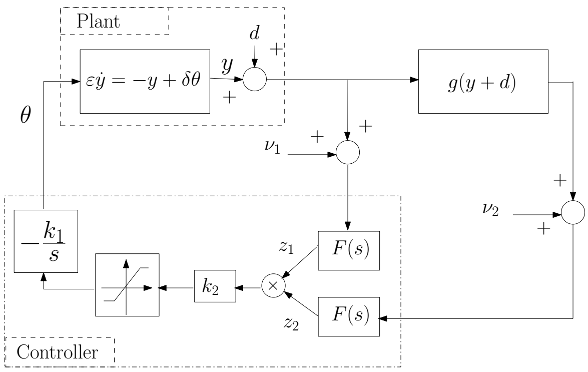

Figure 1. The dynamic extremum seeking scheme.

It can be shown that the global result of Theorem 1, which holds with $F(s) = s$ and $\nu_1 = \nu_2 = 0$, becomes semiglobal and with a larger bound on the maximal distance $y - y^\star$ when measurement noise is present and the filter

$$F(s) = \frac{s}{(\tau_1 s + 1)^n}, \tag{4}$$

with $n \geq 2$, is considered in place of the ideal derivative. The aim of this filter is twofold and is a key ingredient of the scheme: it has to estimate the time derivatives of the input and the output of $g(\cdot)$, to resemble

[1] The set $\mathcal{B}(r_0, r)$ denotes the ball centered in r_0 with radius r.

the filters in the ideal case, and it has to filter out the measurements noises ν_1 and ν_2. To match these requirements, it is necessary that a "frequency" separation between (at least some components of) the signal $d(t)$ and the measurement noises exits.

Note that in this framework we consider the output of the plant as a simple first order linear dynamical system plus a signal $d(t)$ which takes into account disturbances, plant nonlinearities and model approximations. To give an idea, with respect to the FTU facilities, the transfer function that links y, the plasma's horizontal position, with θ, can be approximated by a first order linear model, whereas the signal $d(t)$ models the plasma horizontal fluctuations induced by the actuator nonlinearities and disturbances. Those fluctuations are then considered by the controller to retrieve the minimum of $g(\cdot)$, *i.e.*, the signal $d(t)$ is considered as the "virtual" probing signal in terms of the classical extremum seeking approach [8; 11; 4]. These considerations lead to choosing the filter parameters so that $F(s)$ approximates a time derivative action in the frequency range of (the useful components of) $d(t)$, *i.e.* when $\omega << 1/\tau_1$, and it is a low-pass filter at higher frequencies so as to filter out the measurement noise by tuning the value of $n \geq 2$ and resulting in a sharp band-pass filter.

Note that unlike the classical extremum seeking scheme, the "probing" signal $d(t)$ does not need to be sinusoidal but simply needs to satisfy the persistence of excitation condition (2).

3 Tuning the extremum seeking parameters

In this section, we show by means of two different examples how the control parameters k_1, k_2, τ_1, and n can be chosen to drive y close to $y^\star$. First note that k_1 multiplies the filter outputs $z_1(t)$ and $z_2(t)$ after the saturation, so it is associated with the large signal behavior and also corresponds to the maximum time derivatives of $\theta(t)$. Conversely, k_2 multiplies the signals before the saturation, so it is associated with the small signal behavior and its effect is negligible for large signals where the saturation is active. k_2 can then be seen as the square root of the static gain of $F(s)$. Increasing values of the integer parameter n increase the steepness of the Bode diagram for $\omega > 1/\tau_1$, resulting in a stronger low-pass action.

3.1 First example

In this first example we consider the unknown function $g(y) = (y-4)^2$, $y(0) = 0$, $\varepsilon = 0.01$, $\delta = -2$, $d(t) = 0.05\sin(\sqrt{2}t) + 0.02\sin(30\sqrt{3}t)$ and no measurement noise, $\nu_1 = 0$ and $\nu_2 = 0$.

Since no noise is affecting the system, we may select the filter to resemble a time derivative as much as possible, so we set $\tau_1 = 10^{-4}$. With this choice, the filter is able to approximate the derivative of the two terms of $d(t)$. The simulation results are shown in Figure 2 for different values of $k_1 = \{1, 2, 10\}$ and fixed $k_2 = 1$, whereas in Figure 3 $k_2 = \{0.1, 1, 10\}$ and $k_1 = 1$. It is clear the following role of the two

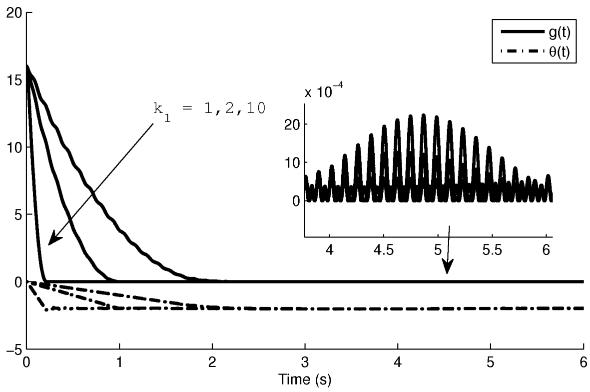

Figure 2. First example: simulation results. $g(y(t))$ (solid) and $\theta(t)$ (dash-dotted) $k_1 = \{1, 2, 10\}$ and $k_2 = 1$.

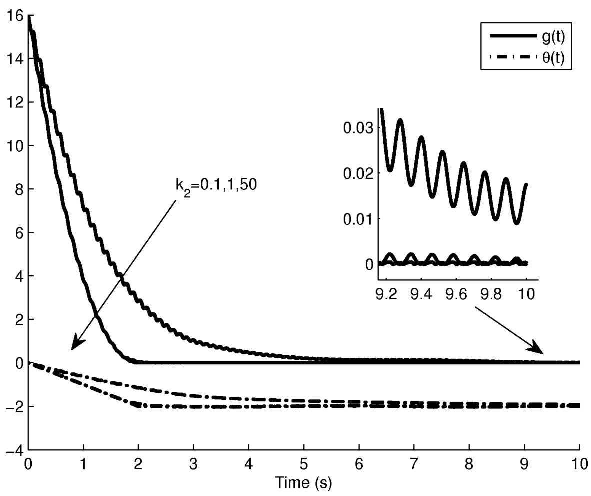

Figure 3. First example: simulation results using $k_1 = \{1, 2, 10\}$ and $k_2 = 1$.

gains k_1 and k_2: k_1 strongly increases the convergence rate of y to $y^\star$ and $|\dot{\theta}| \leq k_1$, whereas k_2 acts more like a "magnifier" to converge to the minimum when z_1 and z_2 are small. Generally, the greater ε is, the smaller k_1 should be. This is also suggested by the bound $\mathcal{A} = \mathcal{B}(y^\star, 2\varepsilon k_1 + \bar{d})$ in Theorem 1.

It is also interesting to analyze the case with $\tau_1 = 0.05$ depicted in Figure 4, with $k_1 = k_2 = 1$. In this case the filter does not perform a sufficient approximation of $\dot{d}(t)$, then the feedback system induces oscillations which themselves are interpreted by the controller as the "probing signal". Those oscillations have lower frequency than those of d, and the filter is able to perform a slightly better approximation of their derivative. Therefore, the system starts to converge towards the minimum. Certainly, the amplitude and the frequency of those oscillations depend critically on the value of ε and k_1, as shown in Figure 5 for $k_1 = 10$. This is an interesting property of this approach: any signal which is feed into the filter as long as its time derivative can be approximated with sufficient precision, can be regarded as an eligible "probing" signal.

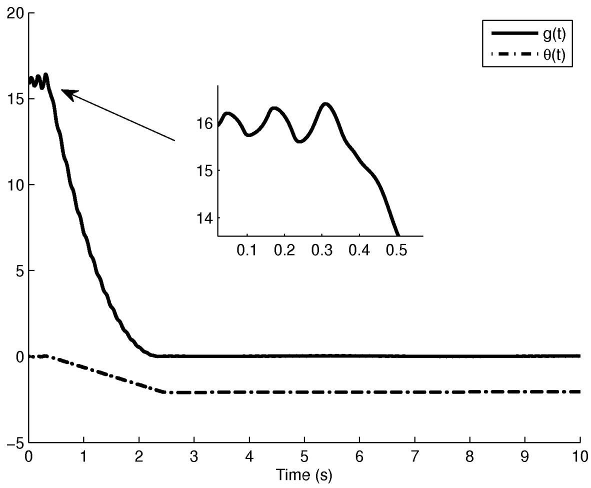

Figure 4. First example: simulation result with $\tau_1 = 0.05$ and $k_1 = k_2 = 1$.

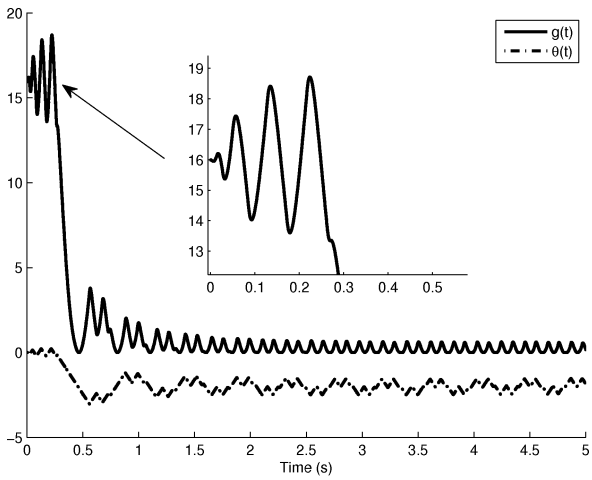

Figure 5. First example: simulation result with $\tau_1 = 0.05$, $k_1 = 10$ and $k_2 = 1$.

3.2 Second example

In the second example we consider a measurement noise given by

$$\nu_1(t) = w_1(t) + 0.05\sin(60t), \quad (5)$$

$$\nu_2(t) = w_2(t) + 0.05\sin(150t), \quad (6)$$

where w_1 and w_2 are band-limited white Gaussian noises (as implemented in Matlab) with zero mean and power $2e-5$. Note that the second component of $d(t) = 0.05\sin(\sqrt{2}t) + 0.02\sin(30\sqrt{3}t)$ has almost the same frequency of the sinusoidal component of the noise ν_1. However, we may select $\tau_1 = 0.01$ obtaining a good approximation of the derivative of the first component of $d(t)$ and filtering out what remains. Simulation results are shown in Figure 6 for $n = \{2, 4\}$, $g(y) = (y-4)^2$, $y(0) = 0$, $\varepsilon = 0.01$, $\delta = -2$ and $k_1 = 2$, $k_2 = 1$. Increasing the value of n helps to filter out the noise enhancing the closed loop performance.

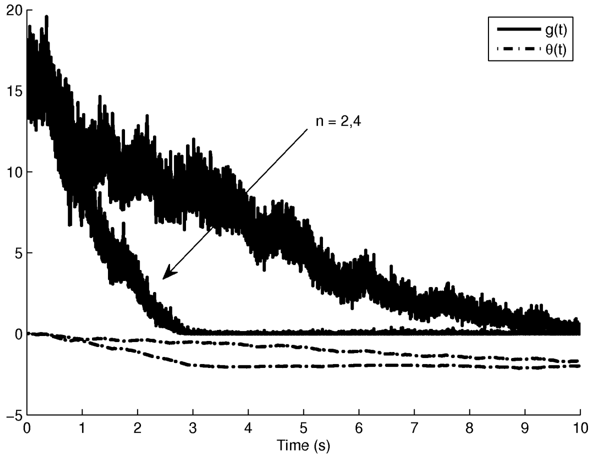

Figure 6. Second example: increasing the value of n from 2 to 4.

In another set of tests reported in Figure 7, we try to exploit the self excitation property discussed in the previous example, but when the measurement noise is nonzero. In particular, we select $\tau_1 = 0.05$ and we show the results for different values of $n = \{2, 4, 6\}$. This time, increasing n reduces the performance of the system: this is due to the fact that if n increases, the approximation of the derivative becomes worse for signals with frequency around the value of $1/\tau_1$ because of the multiple poles in $1/\tau_1$, and with the selected gain k_1 and $\varepsilon = 0.01$, the frequency of the oscillations induced by the closed loop system are close to this value as well.

Finally, in Figure 8 we show how decreasing k_1 from 2 to 1, with $n = 6$, increases the performance of the system for large times because the oscillations are reduced. This improvement can be explained by two reasons: the bound given on Theorem 1, and the fact that the self oscillations have slower frequency and the filter can approximate better their time derivatives.

4 Conclusions

In this paper we conveyed how the controller parameters of the new extremum seeking scheme proposed in [2] can be selected to induce desirable closed-loop performance. It has been shown how the filter can be

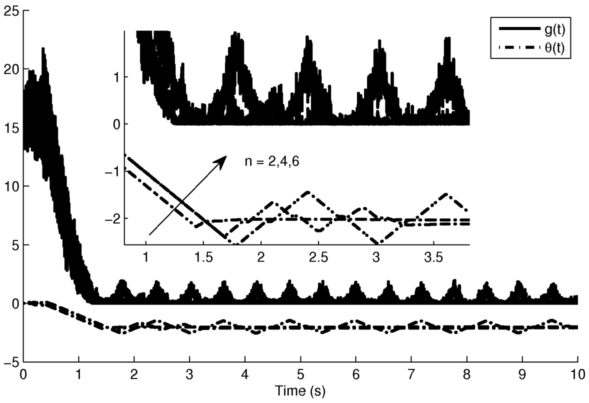

Figure 7. Second example: increasing n reduces the performance of the system.

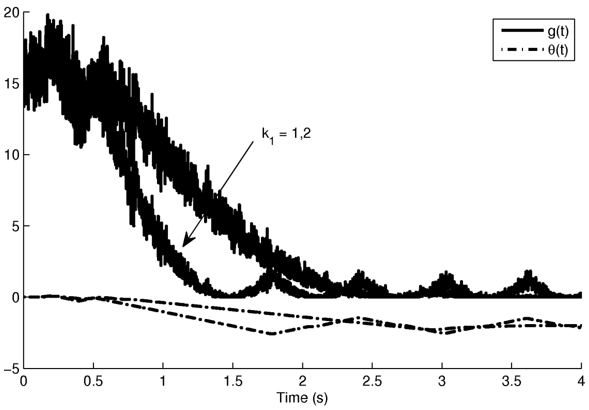

Figure 8. Second example: decreasing k_1 from 2 to 1 to get closer to the minimum dealing with small $1/\tau_1$ and $n = 6$.

chosen depending on the property of the disturbance affecting the nonlinearity input signal and of the noise affecting the measurements. The relation between the controller gains k_1, k_2 and the filter parameter τ_1 has been discussed, highlighting the property of self excitation which may lead to improved convergence even when the approximated derivative of the signal $d(t)$ can hardly be evaluated.

References

[1] K.B. Ariyur and M. Krstic. *Real-Time Optimization by Extremum-Seeking Control*. Wiley-Interscience, 2003.

[2] D. Carnevale, L. Zaccarian, A. Astolfi, and S. Podda. Extremum seeking without external dithering and its application to plasma rf heating on ftu. In *Proc. 4tth IEEE Conf. Decision and Control, Cancun, Mexico*, pages 3151–3156, 2008.

[3] C. Centioli, F. Iannone, , M. Panella, L. Pangione, S. Podda, V. Vitale, and L. Zaccarian. Advances in the experimentation of extremum seeking techniques to maximize the RF power absorption in FTU. In *Symposium on fusion engineering (SOFE)*, Knoxville (TN), USA, September 2005.

[4] C. Centioli, F. Iannone, G. Mazza, M. Panella, L. Pangione, S. Podda, A. Tuccillo, V. Vitale, and L. Zaccarian. Maximization of the lower hybrid power coupling in the Frascati Tokamak Upgrade via extremum seeking. *Control Engineering Practice*, 2008, to appear.

[5] C.S. Drapper and Y.T. Li. Principles of optimalizing control systems and an application to the internal cobustion engine. *ASME*, 160:1–16, 1951.

[6] M. Guay, D. Dochain, and M. Perrier. Adaptive estremum seeking control of continuous stirred tank bioreactors with unknown growth kinetics. *Automatica*, 40:881–888, 2004.

[7] JB Lister, A. Portone, and Y. Gribov. Plasma control in ITER. *Control Systems Magazine, IEEE*, 26(2):79–91, 2006.

[8] M.Krstić and H.H.Wang. Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36:595–601, 2000.

[9] I.S Morosanov. Method of extremum control. *Automation and Remote Control*, 18:1077–1092, 1957.

[10] K.S. Peterson and A.G. Stefanopoulou. Extremum seeking control for soft landing of an electromechanical valve actuator. *Automatica*, 40:1063–1069, 2004.

[11] Y. Tan, D. Nešić, and I.M.Y. Mareels. On non-local stability properties of extremum seeking control. *Automatica*, 42:889 – 903, 2006.

[12] V. Vitale. A 10khz feedback control system for plasma shaping on FTU. In *11th IEEE NPSS Real Time Conference*, June 1999.

[13] L. Zaccarian, C. Centioli, F. Iannone, M. Panella, L. Pangione, S. Podda, and V. Vitale. Optimization of RF power absorption by optimization techniques using the Lower Hybrid current drive of FTU. *Fusion Engineering and Design*, 74(1-4):543–548, 2005.