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ANGULAR RATE SENSORS BASED ON THE MEMS RING RESONATORS

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Abstract

Principles of construction of a micromechanical angular-rate sensor on the basis of the wave solid-state gyro with the ring resonator and positional excitation are observed. The special attention is given to implementation of augmented autogenerating excitation of the resonator, implementation of a compensating principle of the measurement, specific inaccuracies of the sensor, to the arrangements supplying linearity of transformation of measured angular rate. The derived estimations of the basic informational parameters of the sensor determine its primary application for a high dynamic vehicles demanding measurement of large angular rates, a small time of its readiness, wide bandwidth of the sensor.

Key words

Sensor, micromechanics, angular rate, solid-state, ring, MEMS

1 Introduction

Wave solid-state gyroscopes based on ring resonators are currently well represented in micromechanical configuration with silicone MEMS [1, 2, 3, 4]. With positional excitation of the resonator when profile of radial forces exciting resonator is rigidly connected to device body and corresponds to main elliptic oscillation mode, such gyroscopes operate in angular rate sensor mode [5, 6]. Self-maintained excitation of resonator and implementation of compensatory measurement principle enable obtaining micro angular velocity sensors up to ±500°/sec with readiness time 0.1÷0.3 sec and bandwidth 30÷50 Hz. Similar characteristics are also provided by devices with metal resonators of cylindrical shape which theory is close to that of ring resonators [6, 7]. Presented characteristics of angular velocity sensors ensure quite large area of their practical application.

Micromechanical vibrating ring gyroscopes can be used for the short time inertial navigation or as a part of the navigation system which is corrected from external positional system. Other numerous applications are connected with ride stabilization and roll-over detection, some consumer electronic applications such as stabilization of pictures in digital video camera and inertial mouse in computers; robotics applications; and, a wide range of military applications such as guidance of missiles and platform stabilization.

2. Construction and principle of operation

In Fig. 1 the kinematics scheme of angular velocity sensor based on ring resonator with minimum sufficient number of electrodes for ensuring compensatory measurement principle is presented. In this scheme the resonator 1 by means of the eight miniature elastic elements 3 is suspended in the case of the device 2.

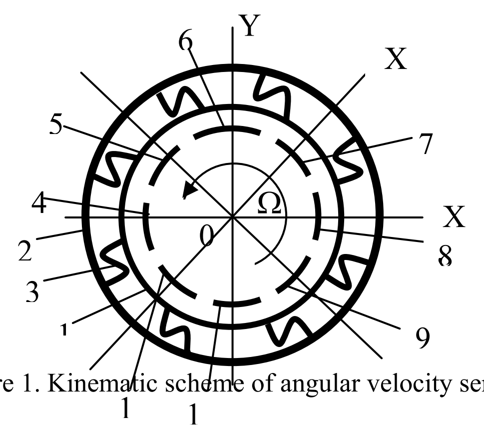

Figure 1. Kinematic scheme of angular velocity sensor

The flexible suspension of resonator should deliver device body rotation Ω to resonator, ensure minor radial and tangential displacements of resonator points during its oscillations, slightly affect to the main resonator characteristics. Electrical electrodes 4-11 isolated from each other are rigidly connected to device body. Thus, electrodes 4, 8 provide positional electrostatic excitation of the resonator, electrodes 6, 10 work in a control loop of autogenerating excitation, electrodes 7, 11 form the capacitor converters of output radial displacements, electrodes 5, 9 form electrostatic forces generators.

The principle of micromechanical solid-state wave gyroscopes operation is based on inert properties of elastic waves. The form of fluctuations when there's no rotation of the device case (Ω =0) is shown on the left side of the Fig. 2. The ring form is near to the elliptic form when the ring is deformed. The radial movings of a ring are near to zero in the points 1, 2, 3 and 4. This points located on the axes that is turned through the angle φ=π/4.

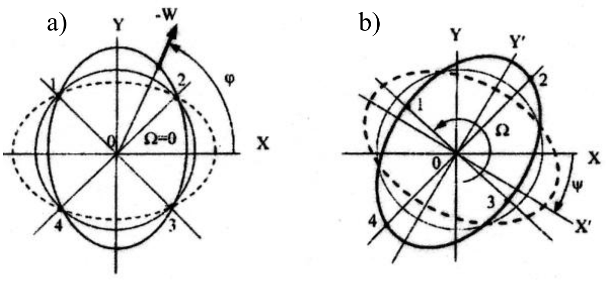

Figure 2. Oscillations of the ring resonator:
a) no rotation of sensor
b) sensor is rotated in the plane of the picture

3. Mathematical description

Assuming that specific density of radial forces acting on resonator from suspension elements is significantly smaller than that of radial forces generated by excitation system, resonator dynamic equation is as follows:

$$\ddot{w}'' - \ddot{w} + 4\,\Omega\,\dot{w}' + 2\dot{\Omega}w' + \xi_1(\dot{w}^{\mathrm{VI}} + 2\dot{w}^{\mathrm{IV}} + \dot{w}^{II}) + \chi^2(w^{\mathrm{VI}} + 2w^{\mathrm{IV}} + w^{II}) = f\,(t,\varphi), \quad (1)$$

where $w(\varphi,t)$ is the radial displacements of the ring point located angularly φ relatively of the center line.

Parameters χ^2 and ξ_1 describe elastic and damping forces for rectangular cross-section of the ring and can be calculated by the formulas:

$$\chi^2 = \frac{Eh^2}{12\rho R^4}, \quad \xi_1 = \frac{\xi h^2}{12\rho R^4} \quad (2)$$

where E, ρ are the elasticity modulus and the ring material density, R and h are the center line radius and the thickness of ring, ξ is the coefficient of viscous friction forces.

In general case, density of radial forces reduced to dimensionality of linear acceleration and created by excitation system is determined by dependency [5],

$$f(t,\varphi) = \pm\frac{\varepsilon_0}{2d_0^2 h\rho}[V^2(t,\varphi)]'' \;, \quad (3)$$

where d_0 is the clearance between electrode and resonator; $\varepsilon_0 = 8{,}85\cdot 10^{-12}$ *F/m* is dielectric constant; signals minus and plus correspond to external or internal location of electrodes; $V(t,\varphi)$ is excitation voltage. If voltage with V_0 amplitude and frequency equal to half of resonator free oscillations is supplied to two excitation electrodes 6, 10 with resonator coverage angle $2\beta_0$, function of positional resonance force (3) results in

$$f(t,\varphi) = f_0 \cos\nu t \cos 2\varphi, \quad (4)$$

where $f_0 = \dfrac{2\varepsilon_0 V_0^2 \sin 2\beta_0}{\pi d_0^2 \rho h}$.

Further analysis into resonator requires determination of parameters χ and ξ1. For that end with Ω=0, $f(t,\varphi)=0$, particular solution of resonator equation for initial conditions $w(0,0)=w_0$, $\dot{w}(0,0)=0$ is as follows

$$w(t,\varphi) = w_0 e^{-\delta\nu_c t}\left(cos\nu_{1c}t + \frac{\delta}{\sqrt{1-\delta^2}} sin\nu_{1c}t\right)cos2\varphi \quad (5)$$

where δ is a damping coefficient of free oscillations, ν_c И ν_{1c} are the frequencies of free undamped and damped oscillations.

Substituting solution (5) into equation (1) and dividing oscillation modes the following is found

$$\nu_c = \frac{6}{\sqrt{5}}\chi = \frac{6}{\sqrt{5}}\frac{h}{R^2}\sqrt{\frac{E}{12\rho}}[s^{-1}],$$
$$\nu_{1c} = \nu_c\sqrt{1-\delta^2},$$
$$\xi_1 = \frac{5}{18}\delta\nu_c = \frac{5\nu_c}{36D}, \quad (6)$$

where D is a resonator Q-factor.

Detailed analysis of quasi-stationary modes of wave solid-body gyroscopes with parametric excitation is given in the papers [5, 6, 7, 8]. The steady errors stipulated by a various structural and technological factors by methods of Bubnov-Galerkinare were estimated for gyroscope, which operates in integrating mode.

However, for gyroscopes, which operate in the mode of angular velocity sensors, the requirements for device short readiness time and required bandwidth or short duration of control time are of a priority. To obtain evaluation of these parameters the following procedure for investigation of resonator static and dynamic characteristics may be used. Assuming that own frequencies of a free resonator and forced oscillations coincide for gyroscope self-maintained resonance excitation, the general solution of equation (1) may be presented as

$$w(t,\varphi) = [a(t)cos2\varphi + b(t)\sin 2\varphi]cos\,\nu t + $$
$$+ [m(t)cos2\varphi + n(t)\sin 2\varphi]\sin \nu t. \qquad (7)$$

where *a(t), b(t), m(t), n(t)* are undetermined functions of the time.

These functions determine the dynamics of transformation of the ring oscillations from initial state to the finish state.

Substituting solution (7) into equation (1) and combining components of the calculated expression with identical trigonometric functions, it is possible to write:

$$\ddot{a} = (\nu^2 - \frac{36}{5}\chi^2)a - \frac{36}{5}\xi_1\dot{a} + \frac{8}{5}\Omega\dot{b} - \frac{36}{5}\xi_1\nu m -$$
$$-2\nu\dot{m} + \frac{8}{5}\Omega\nu n - f_e + \frac{4\dot{\Omega}b}{5},$$
$$\ddot{b} = -\frac{8}{5}\Omega\dot{a} + (\nu^2 - \frac{36}{5}\chi^2)b - \frac{36}{5}\xi_1\dot{b} - \frac{8}{5}\Omega\nu m -$$
$$-\frac{36}{5}\xi_1\nu n - 2\nu\dot{n} - \frac{4\dot{\Omega}a}{5} - f_k,$$
$$\ddot{m} = \frac{36}{5}\xi_1\nu a + 2\nu\dot{a} - \frac{8}{5}\Omega\nu b + (\nu^2 - \frac{36}{5}\chi^2)m - \qquad (8)$$
$$-\frac{36}{5}\xi_1\dot{m} + \frac{8}{5}\Omega\dot{n} + \frac{4\dot{\Omega}n}{5},$$
$$\ddot{n} = \frac{8}{5}\Omega\nu a + \frac{36}{5}\xi_1\nu b + 2\nu\dot{b} - \frac{8}{5}\Omega\dot{m} +$$
$$+(\nu^2 - \frac{36}{5}\chi^2)n - \frac{36}{5}\xi_1\dot{n} - \frac{4\dot{\Omega}m}{5}.$$

where f_e is the excitation force, which is used for autogenerating of primarily oscillation.

For a gyro of compensating type it is necessary to consider in additional force

$$f_k = f_{k0} \cdot k_1 n. \qquad (9)$$

It should be noted that in resonance mode $\nu^2 - \frac{36}{5}\chi^2 = 0$.

4. Results of simulation

For investigation of the properties of the ring gyroscopes it is used equations (8). The time of simulation is divided into two parts. From t=0 to t= 0.5 sec angular rate of the device case Ω=0. After this time the device case is rotated with angular rate Ω = 300 deg/sec.

From t=0 excitation force f_e is applied and primarily oscillations are developed. For realizing autogenerating forced regime this excitation force is

$$f_e = 0.2 f_0 sign(\dot{\alpha} + \nu m), \qquad (10)$$

where f_0 =8V during the first 0.08 sec of simulation and thereafter f_0 =5V. This approach provides forced regime of primarily oscillations generation.

In Fig. 3 shows results of integration of system (5) for a direct transformation mode. In the processes of simulation the follows parameters of micromechanical sensor are used:

$$R = 3\cdot10^{-3}\text{m},\ h = 0.15\cdot10^{-3}\text{m},\ E = 1{,}96\cdot10^{11}\frac{N}{m^2},$$

$$\rho = 2.3\cdot10^3\frac{kg}{m^3},\ \beta_0 = 10^\circ,\ d_0 = 4\cdot10^{-6}m,$$

$$D = 10000,\quad \Omega = \pm300^\circ/\sec.$$

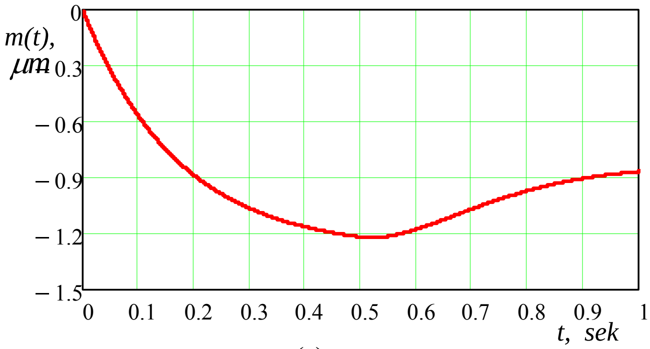

Figure 3. Time response $m(t)$ for a direct transformation mode.

Results of simulation show that at high Q-factor of the resonator with dominance in 4-5 order of magnitude the following inequalities are came true:

$$|m(t)| >> |a(t)|;\quad |n(t)| >> |b(t)|;$$
$$|\dot{a}(t) + m(t)\nu| >> |-a(t)\nu + \dot{m}(t)|. \qquad (11)$$

For this reason, here will be analyzed functions $m(t)$ and $n(t)$ only. The function $m(t)$ shows that the processes of oscillation excitation from zero to steady state value - 1.2 μm required 0.5 sec. This time determines an available time of a gear for measurement of angular rate of the device case. Excitation voltage is equal f_0 =5V in all time of excitation in this experiment.

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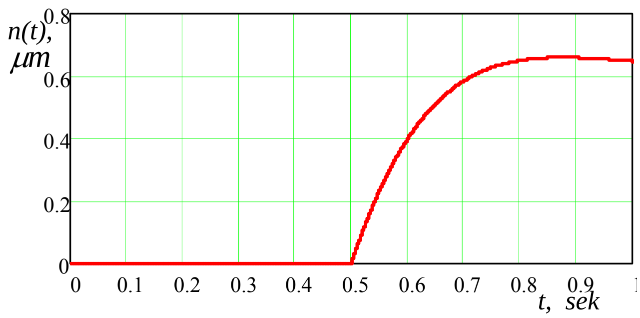

Figure 4. Time response $n(t)$ for a direct transformation mode.

Besides, processes of measurement have inadmissible delaying which is determined by the function $n(t)$ from the start of the device case rotation t=0.5 sec to approximately 1 sec (see Fig. 4).

Rotation of device case ($\Omega \neq 0$), produces a big angle of ellipse rotation and reduce the linear displacement of the resonator in an excitation zone.

In steady state mode $a = b = 0$ and the ellipse angle turn for $\Omega = 300\ ^0/\text{sec}$ will be

$$\psi_{ss} = 0.5arctg\frac{n}{m} = -0.5arctg\frac{2\Omega}{9\xi_1} \cong -18^0 .$$

For exclusion of mentioned imperfections it is offered to use gyroscope of compensating type. For a gyro of compensating type it is necessary to input additional force (9).

For reduction of the readiness time, it is necessary to increase the voltage of excitation during the readiness time.

As it is possible to see in Fig. 5, for $f_k = 1 \cdot 10^8$ V/m and f_0 =8V, the processes of oscillation excitation from zero to steady state value -1.2 μm required 0.08 sec. Recall, that the time of excitation of oscillation determine the readiness time of sensor.

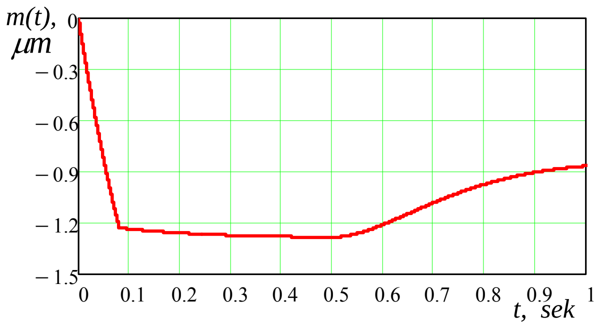

Figure 5. Time response $m(t)$ for a compensating mode.

Fig. 6 shows, that the delay of ellipse axis deflection, as reaction on appearance of angular rate Ω in instant t=0.5 sec, is significantly smaller, then in the case of a direct transformation mode (see Fig.. 4).

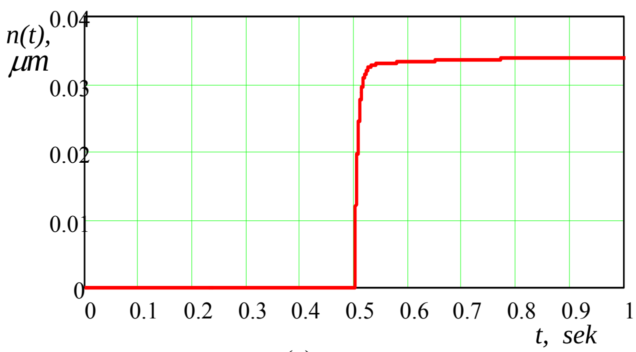

Figure 6. Time response $n(t)$ for a compensating mode.

Output value of the sensor in volts can be calculated as $V = k_1 n$.

The function V(t) in neighbourhood of t=0.5 sec in Fig. 7 is presented. The time delay for this case is approximately 0.03 sec. Transformation scale factor from angular rate to the voltage is K_{TR} =3.45V/300deg/sec=11.5 mV/deg.sec.

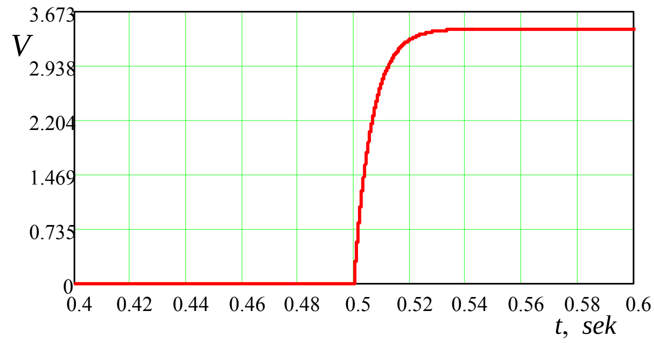

Figure 7. Output sensor's voltage.

Evolution of the elliptic vibrations for constant angular rate for several moments of time is shown in fig. 8. Real radial displacements of the elliptic vibrations are increased in 10 times for obviousness.

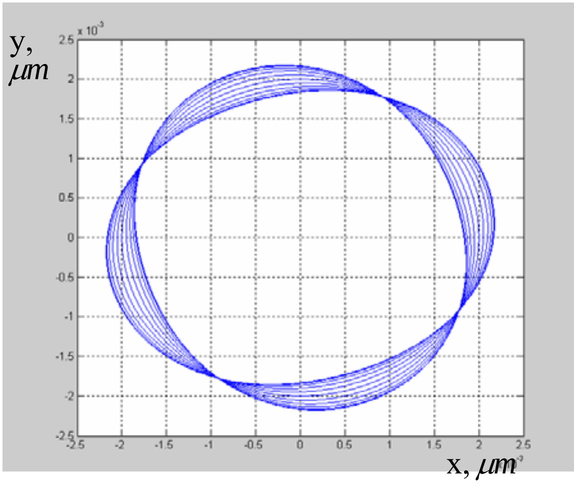

Figure 8. Flexible ring vibration.

4. Pass band of angular velocity sensor

In the real conditions it is necessary to take into account the angular rate of the device case and it angular acceleration.

In this situation according to equations (8) the device case angular rate $\Omega(t)$ and the first derivative $\dot{\Omega}(t)$ are inputted as parameters. These parameters are multiplied with the components of state vector.

Nonstationarity of the equations (8), and also the multiplicativity of the measured parameter $\Omega(t)$ and its derivative with the components of state vector do not allow using the method of transfer functions and frequency characteristics. For this reason experimental comparison of amplitude and a phase input and output signals is a unique practical method of an estimation of required parameters.

Angular rate of the device case in the program in the form of function of a time for various frequencies is modeled. For angular rate the expression $\Omega(t) = \Omega_0 \cos(\omega t)$ is used. In this case for angular acceleration it is possible to write

$$\dot{\Omega}(t) = -\Omega_0 \omega sin(\omega t) .$$

This angular rate and acceleration are actuated in model after the termination of process of excitation (in 0.5 seconds after gear powerup).

For calculation of pass band the method of Bode Diagram is used. For series of frequencies ω, magnitude and phase of steady state oscillations of output variable n(t) is calculated.

All calculations in the program Matlab is fulfilled. After data processing Bode Diagram is plotted. These normalized diagrams for different k_1 are shown in Fig. 9.

The analysis shows that these frequency responses are close to frequency responses of an aperiodic transfer functions with a transfer function

$$W(s) = \frac{1}{Ts+1}. \qquad (12)$$

Thus, the transfer function (12) can be used as dynamic model of the gyroscopes based on ring resonators of compensating type. Time constant T and a pass band depend on depth of negative feedback in automatic compensation loop.

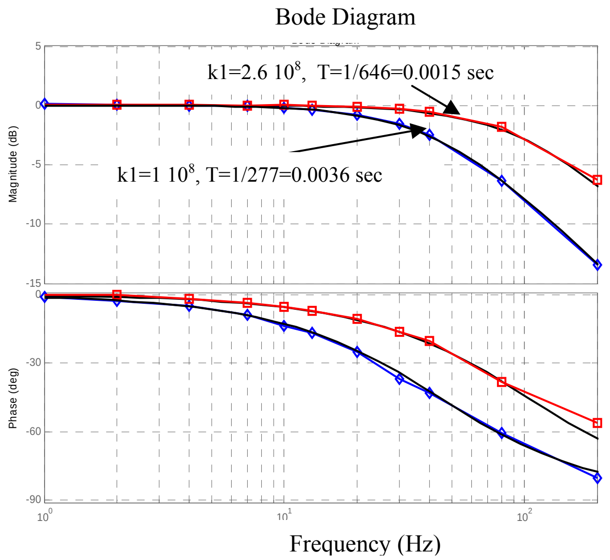

Figure 8 Bode Diagrams for wave solid-state gyroscopes of compensating type.

Conclusion

1. Equations (8) allow investigating dynamical properties of envelope curve for oscillations of ring resonator subject to the device case angular rate and angular acceleration.
2. It is shown, that the wave solid-state gyros with the ring resonator and a direct transformation of ellipse rotation have inadmissible delay.
3. Utilization of compensation signal allows decreasing delay of measured angular rate. Augmentation of the coefficient k_1 leads to depth of negative feedback in automatic compensation loop and leads to expansion of pass band.

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