EXPERIMENTAL INVESTIGATION OF ADAPTIVE SYNCHRONIZATION OF TWO RÖSSLER CIRCUITS

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Abstract

In this work we investigate synchronization in an experimental setup consisting of two diffusively coupled electronic chaotic circuits where the coupling gain is controlled by an adaptive law. We show that an adaptive law, which assumes that the gain changes are proportional to the error between the states of the two circuits, suffers from the presence of unavoidable tolerances in the circuit parameters. For this reason, we introduce a new adaptive law which on the contrary is able to handle the problem in the presence of circuital mismatches.

Key words

Synchronization, adaptive law, chaotic circuits.

1 Introduction

Synchronization is a collective behavior commonly found in many natural, social and man-made systems [Boccaletti, 2008]. It emerges through the interaction of two or more nonidentical dynamical units that, as a consequence, show a collective state in which they behave in a coordinated way. Synchronization may be attained in many different ways, and it is, for instance, commonly observed in diffusively coupled units. In general, when synchronization of units diffusively coupled into a complex network is dealt with, the coupling gain is assumed to be fixed in time. However, in many real-world networks links evolve in time as a function of the environmental changes or external stimuli [Boccaletti et al., 2006], providing to the network adaptive capabilities to readjust the link weights so that synchronization is attained [De Lellis, 2008]. This has motivated the study of adaptive approaches to synchronization, which can be either global, when the same adaptive coupling gain for all network edges is used [Li et al. 2008], or local/decentralized, when each edge has its own adaptive gain [De Lellis, 2008].

In local adaptive strategies, the adaptive gain of each edge usually evolves as a function of some measure of the difference between the state of the nodes that it links. Different rules have been proposed for the dynamics of the adaptive gain. A simple and commonly used form is to set the rate of change of the gain to be proportional to the norm of the difference of the state vectors or of the node output functions [De Lellis, 2008; Sorrentino and Ott, 2008; Shafi and Arcak, 2015]. In such cases, synchronization is attained while the adaptive coupling gains settle to constant values. Adaptive strategies able to adjust not only the network gains but also the network structure have been introduced [DeLellis et al., 2010]. Interestingly, the network topology emerging from the application of such strategies shows different properties as a function of the interplay between the dynamical process occurring on the network and the dynamical evolution of the network itself [Avalos-Gaytn et al., 2012]. Adaptive synchronization can also be attained in the case of slowly evolving network topologies and in the presence of time delays in the interconnections by using, for instance, the approaches introduced in [Sorrentino and Ott, 2008] and in [Selivanov et al., 2012], respectively.

In this paper, we consider the general form assumed for the adaptive gain dynamics and apply it to experimentally synchronize two electronic chaotic circuits mimicking the Rössler oscillator. The practical application of the adaptive law leads to some challenging issues. In fact, the unavoidable tolerances between the circuits make impossible identical synchronization (that is, a synchronization error exactly equal to zero), whereas a weaker form of synchronization, with a bounded error, is still feasible. We show that the form usually adopted for the coupling gain cannot be directly applied to our case study and introduce a new one able to attain adaptive synchronization in our experimental setup.

The rest of the paper is organized as follows. In Sec. 2 the experimental setup is introduced. In Sec. 3 adaptive synchronization in the experimental setup is investigated. Sec. 4 concludes the paper.

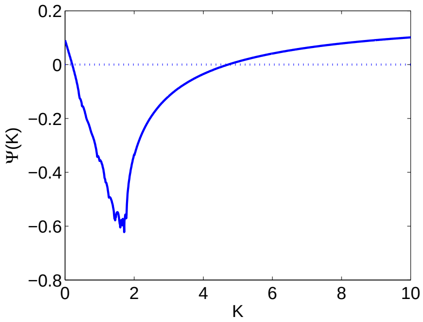

Figure 1. Master stability function for the Rössler system coupled as in Eq. (1).

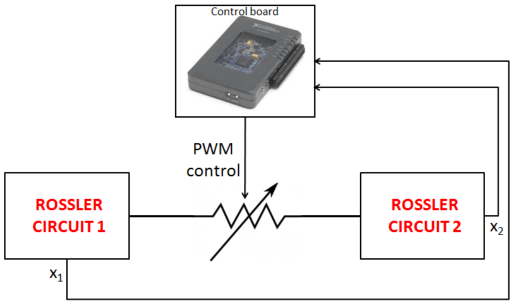

Figure 2. Block scheme of the experimental setup.

2 Experimental setup

We consider two Rössler circuits coupled through the variable x, as follow:

$$\begin{cases} \dot{x_1} = -y_1 - z_1 + \sigma(x_2 - x_1) \\ \dot{y_1} = x_1 + ay_1 \\ \dot{z_1} = b + z_1(x_1 - c) \\ \dot{x_2} = -y_2 - z_2 - \sigma(x_2 - x_1) \\ \dot{y_2} = x_2 + ay_2 \\ \dot{z_2} = b + z_2(x_2 - c) \end{cases} \quad (1)$$

where a, b and c are system parameters, regulating the dynamical behavior of each circuit. The circuits are realized following the scheme reported in [Buscarino et al., 2014]. According to that approach, the dynamics is rescaled in time, in our case the rescaling factor has been fixed equal to $\tau = 5.5ms$. The experiments were carried out on two circuits with $a = 0.2$, $b = 0.2$ and $c = 9$, in order to assure chaotic behavior. Although the parameters are nominally identical for the two circuits, the use of standard off-the-shelves components with $\pm 5\%$ tolerance makes them slightly different. We will show that the mismatch between the two circuits plays a key role in the choice of the adaptive law to be used.

Among the possible coupling configurations for two Rössler systems, we selected the one of Eqs. (1) as it leads to a master stability function of type III [Huang et al., 2009]. This represents the most interesting case as the largest transverse Lyapunov exponent (which is negative when the synchronous state is stable) depends on the coupling coefficient in a non-trivial way. In fact, as shown in Fig. 1, $\Psi(K)$ is negative only in an interval of $K = 2\sigma$ delimited by two threshold values.

In the circuit implementation of system (1) the state variables are associated to voltages across capacitors so that, to obtain a constant diffusive coupling between them, it would be sufficient to insert a single resistor between the two capacitors storing the coupled state variables. However, in our experiments the coupling is not constant but varies in time according to an adaptive law of the general form:

$$\dot{\sigma} = f(e) \quad (2)$$

with $e = x_2 - x_1$. Instead of using digital resistors, which provide specific ranges of variation with a limited resolution, in this work, the time-varying coupling is implemented by an analog switch in series with a fixed resistor, driven by a piece-wise-modulated (PWM) signal. To implement such a coupling scheme, two components are necessary: an analog switch (in this study we chose an ADG452), and a control board which calculates the error signal and generates the PWM signal. The ADG452 embeds four independently selectable bi-directional switches, it has a low on-resistance (in the order of 5Ω), fast switching times ($t_{ON} = 70ns$, $t_{OFF} = 60ns$), and it is TTL-/CMOS-compatible. A block scheme of the whole experimental setup is shown in Fig. 2.

For the generation and control of the PWM signal, a National Instruments myRIO board has been used. This is a reconfigurable I/O device equipped with a dual-core ARM® Cortex^{TM}-A9 real-time processing unit and a Xilinx Zynq-7010 FPGA with 28000 programmable cells [National Instruments webpage]. It is able to acquire up to 10 analog inputs and provide 6 analog outputs. Furthermore, a series of built-in functions allows the generation of suitable analog signals. In our approach, the PWM signal is produced as an output by the myRIO and its duty cycle δ is fixed according to the adaptive law (2).

The proposed scheme, in fact, allows to vary the value of the coupling resistor acting on the duty cycle of the PWM signal driving the analog switch. Turning on and off the switch has the effect of multiplying the fixed coupling resistor by a factor inversely proportional to δ, according to the following equation:

$$R_c = \frac{1}{\delta} R_s \quad (3)$$

where R_c is the realized coupling resistance and $R_s = 505\Omega$. In our application, the ADG452 has been controlled with a 4kHz PWM. This choice is also suitable in the final application, since the frequency range of the considered Rössler implementation with the used time rescaling is below 200Hz.

The NI-myRIO acquires the two coupled state variables x_1 and x_2 at a sampling rate of $f_s = 1$kHz. The absolute value of the difference between the two inputs is computed inside the control board which numerically integrates the adaptive law (2), through a fixed step-size Euler integration method with an integration step size of $\Delta t = 0.01s$. The updated value of the coupling strength is realized choosing the suitable value of the duty cycle implementing the corresponding coupling resistance R_c, according to Eq. (3).

3 Experimental analysis

A simple form for Eq. (2) assumes $f(e)$ proportional to the error. Laws of this type have been used in several works with different variants according to the specific form adopted for the error (for instance, absolute error, square error, difference of the whole state vector, or of an output function [De Lellis, 2008; Sorrentino and Ott, 2008; Shafi and Arcak, 2015]). All these functions can be considered to belong to the same class of adaptive laws and without loss of generality we focus on a single member of the class, described by the following relationship:

$$\dot{\sigma} = k|e| \qquad (4)$$

with $e = x_2 - x_1$. The adaptive law in Eq. (4) has been successfully used to synchronize two or more coupled systems in numerical simulations assuming identical dynamics for the units. In particular, in the case of two units, when arbitrary initial conditions are taken into account for the state variables and the initial value of the gain is zero, one observes that the synchronous state is reached while the gain monotonically settles to a constant value which depends on the initial conditions [De Lellis, 2008].

We study the application of the adaptive law (4) for our experimental setup and show that a different scenario appears when the coupled units, as in practical experiments, cannot be made identical. As in [De Lellis, 2008], we consider arbitrary initial conditions for the Rössler circuits and zero for the initial value of the gain. A typical example of the experimental results obtained is shown in Fig. 3, obtained for $k = 1$. As the adaptive law is activated (at time $t \simeq 4s$), the gain increases leading to a stronger coupling which in turns reduces the synchronization error. However, due to the presence of parametric tolerances in the components of the two circuits, a mismatch between the two dynamics is unavoidable. The consequence of this is that the error may be decreased but never exactly annihilated. For this reason, the gain continues to grow up indefinitely. However, as the Rössler circuit has a type III MSF, the continuous increase of the gain drives the system towards the second transition, thus making unstable the synchronous state. In the practice the value of the gain is also limited and it grows up until it saturates to the maximum value. Fig. 3 shows that, when this occurs (for $t > 110s$), the synchronization error is large, due

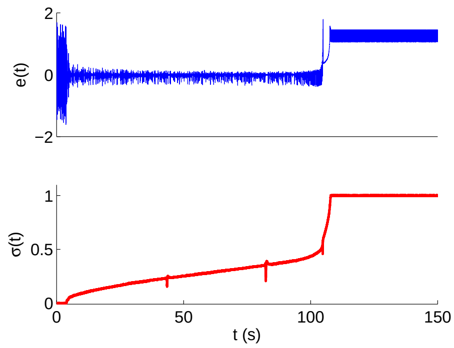

Figure 3. Adaptive synchronization with gain update law as in Eq. (4). Experimental results: trend of the error $x_2(t) - x_1(t)$ and of the gain $\sigma(t)$.

to the fact that the maximum value is beyond the second transition point of the MSF.

A first attempt to overcome the problem is the use of a dead zone. In particular, we consider the following nonlinear function of the error:

$$f_{dz}(e) = \begin{cases} 0 \texttt{ if } |e| < \beta \\ e \texttt{ otherwise} \end{cases} \qquad (5)$$

and incorporate it in the adaptive law:

$$\dot{\sigma} = k|f_{dz}(e)| \qquad (6)$$

so that, when the error is less than β, the gain is maintained constant as $f(e) = 0$.

The parameter β of the dead zone has to be fixed by taking into account the following consideration. When the two circuits are coupled under the best possible conditions, that is, those leading to the smallest synchronization error, this error is still different than zero due to the mismatch between the two circuits. The amplitude of the dead zone has to be larger than this error, so that the rate of change of the coupling in correspondence of its best value is zero. However, this holds for an error which is bounded. We have verified that this condition is not met in our experimental setup and, on the contrary, the occurrence of large error peaks (that is, in the order of magnitude of the range of the state variables) is statistically relevant. For this reason, a slow drift towards the second threshold of the MSF is observed also in this case. This is evident in Fig. 4, obtained for $k = 1$ and $\beta = 0.4V$, and clearly prevents the feasibility of the solution based on the dead zone.

The solution adopted in this paper is to design an adaptive law able to filter the error due to the mismatch between the circuits. Analogously to consensus filters [Olfati-Saber and Shamma, 2005], which in the presence of noise experience a problem similar to what reported in this paper, a new term, $-\alpha\sigma$, is introduced into the adaptive law, so that it has low-pass filtering

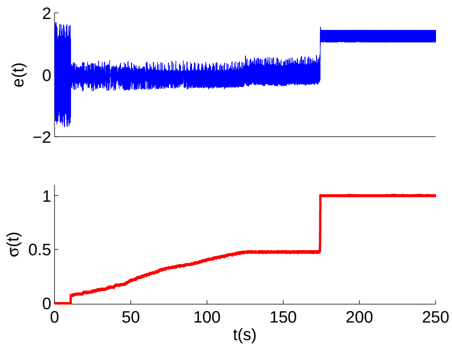

Figure 4. Adaptive synchronization with gain update law as in Eq. (6). Experimental results: trend of the error $x_2(t) - x_1(t)$ and of the gain $\sigma(t)$.

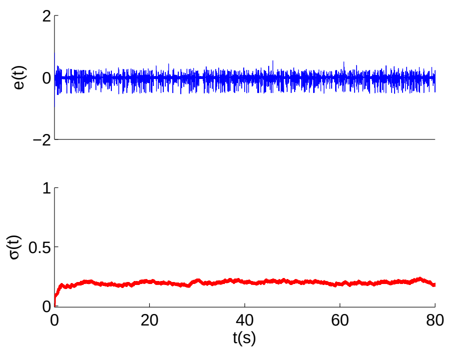

Figure 5. Adaptive synchronization with gain update law as in Eq. (7). Experimental results: trend of the error $x_2(t) - x_1(t)$ and of the gain $\sigma(t)$.

capabilities. The new adaptive law is described by the following relationship:

$$\dot{\sigma} = k|e| - \alpha\sigma \tag{7}$$

where the parameter $\alpha > 0$ is tuned so that to guarantee satisfactory performances of the law. The experimental results, obtained for $k = 1$ and $\alpha = 0.01$ and reported in Fig. 5, show the suitability of the approach. After a fast transient the gain starts oscillating around a steady value, while the synchronization error remains bounded.

4 Conclusions

In this paper an experimental investigation of adaptive synchronization in two coupled Rössler circuits has been dealt with. We have intentionally selected a coupling configuration having a type III MSF. Furthermore, the two circuits present unavoidable parametric tolerances which lead to a mismatch in the two dynamics. Under these conditions, the form of the adaptive law, often adopted in literature, where the analysis is mostly limited to the case of identical systems, cannot be adopted. In fact, we have observed that the mismatch causes a finite, but non-zero erratic difference between the state variables of the two circuits, which in turns may drift the gain towards the second transition in the MSF. To overcome this drawback a new adaptive law, including a further term allowing to incorporate low-pass filtering capabilities in the adaptive law, has been introduced in this paper and experimentally proved to be effective in achieving adaptive synchronization.

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