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STABILITY OF ELASTIC ELEMENTS OF THIN-SHELLED CONSTRUCTIONS UNDER AEROHYDRODYNAMIC ACTION

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Abstract

Dynamic stability of elastic elements of thin-shelled constructions under interaction with flow of fluid or gas is studied. Subsonic regime is considered. Aerodynamic load is determined by asymptotic aerohydromechanics equations [Velmisov, 1986]. The nonlinear model of elastic body is used to problems of aerohydroelastisity. In like statement on base of building the functionals this problems earlier were not researched.

Statements and investigation methods offered for dynamical damping elastic bodies, being in contact with subsonic flow of the fluid or the gas, lead to the study of linked initial boundary problems to systems of partial differential equations. Being based on the construction of functionals, corresponding to these systems, solutions' stability conditions are obtained for some aerohydroelastical problems [Ankilov and Velmisov, 2000; Velmisov and Reshetnikov, 1994], in particular for dynamics of elements of a plane channel, through which fluid flows; elements of profile of a wing; pipeline.

Key words

Aerohydromechanics, aerohydroelastisity, dynamic stability.

1 Stability of elastic elements of wing

Let us consider in more detail the planar problem of aerohydroelasticity about small fluctuations, appearing without a detachment flowing around thin-shelled constructions - the models of wing, which component parts are *n* elastic elements-insertions.

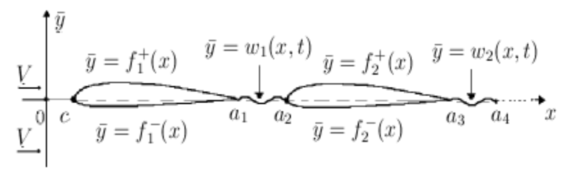

Figure1. Wing profile.

Suppose on plane xOy, in which occur joint fluctuations of elastic insertions and gas, length $[c, d]$ on axis Ox corresponds to wing, and lengths $[a_{2k-1}, a_{2k}]$, $k = 1 \div n$, $-\infty < c \le a_{2k-1} < a_{2k} \le$ $\le a_{2k+1} < a_{2k+2} \le d < +\infty$, $k = 1 \div n-1$ to elastic insertions (fig.1).

In infinitely remote point a velocity of gas is V and has the direction, coinsiding with the direction of axis Ox. Let's indicate: $w_k(x,t)$ and $u_k(x,t)$ $(k = 1 \div n)$ are the function of plate deflections toward axis Oy and Ox correspondingly; $\varphi(x,y,t)$ is the potential of the gas velocity.

Potential of velocity φ satisfies Laplace equation

$$\Delta\varphi \equiv \varphi_{xx} + \varphi_{yy} = 0,\ (x,y) \in G = R^2 \backslash [c,d]\,, \quad (1)$$

border conditions

$$\begin{gathered}\varphi_y^{\pm}(x,0,t) = \lim_{y \to \pm 0} \varphi_y(x,y,t) = f_1^{\pm}(x)\,,\\ x \in (c, a_1)\,,\end{gathered} \quad (2)$$

$$\begin{gathered}\varphi_y^{\pm}(x,0,t) = w_{kt}(x,t) + V w_{kx}(x,t)\,,\\ x \in (a_{2k-1}, a_{2k})\,,\ k = 1 \div n,\end{gathered} \quad (3)$$

$$\begin{gathered}\varphi_y^{\pm}(x,0,t) = f_{k+1}^{\pm}(x)\,,\\ x \in (a_{2k}, a_{2k+1})\,,\ k = 1 \div (n-1)\,,\end{gathered} \quad (4)$$

$$\varphi_y^{\pm}(x,0,t) = f_{n+1}^{\pm}(x)\,,\ x \in (a_{2n}, d)\,, \quad (5)$$

where $f_k^{\pm}(x)$ $(k = 1 \div (n+1))$ are given functions determining the form of undeformable parts of a wing, and condition of unperturbed flow in infinitely removed point

$$|\nabla\varphi|_\infty^2 \equiv \left(\varphi_x^2 + \varphi_y^2 + \varphi_t^2\right)_\infty = 0. \quad (6)$$

Let's present equations of small fluctuations of elastic plates in the manner of

$$\begin{cases} -E_kF_k\left({u_k}'+\frac{1}{2}{w_k}'^2\right)'+M_k\ddot{u}_k+ \\ +g_k\left(t,x,u_k,w_k,\dot{u}_k,\dot{w}_k\right)=0, \\ -E_kF_k\left[{w_k}'\left({u_k}'+\frac{1}{2}{w_k}'^2\right)\right]'+M_k\ddot{w}_k+ \\ +E_kJ_k{w_k}''''+h_k\left(t,x,u_k,w_k,\dot{u}_k,\dot{w}_k\right)= \\ =\rho\left(\varphi_t^+\left(x,0,t\right)-\varphi_t^-\left(x,0,t\right)\right)+ \\ +\rho V\left(\varphi_x^+\left(x,0,t\right)-\varphi_x^-\left(x,0,t\right)\right), \\ x\in\left(a_{2k-1},a_{2k}\right),\quad k=1\div n. \end{cases} \tag{7}$$

Here a prime is used for the derivative with respect to x; a point over letters denotes time derivative; subindexes x, y, t designate partial derivatives with respect to the corresponding variables; ρ is the density of the gas; E_k are the modules of elasticity of plates; F_k are the areas of cross-sections of plates; E_kJ_k are the rigidity of the plates; M_k are the specific mass of the plates; the functions $g_k\left(t,x,u_k,w_k,\dot{u}_k,\dot{w}_k\right)$, $h_k\left(t,x,u_k,w_k,\dot{u}_k,\dot{w}_k\right)$ represent nonlinear components reactions of the basis or other nonlinear influences.

Using methods of the theory of complex variable functions [Lavrentev and Shabat, 1987], the solution of the problem (1) - (6) can be reduced to a system of equations for unknown functions of plates deflections, where right part of the second equation of system (7) will be:

$$\begin{gathered}\rho\left(\varphi_t^+\left(x,0,t\right)-\varphi_t^-\left(x,0,t\right)\right)+ \\ +\rho V\left(\varphi_x^+\left(x,0,t\right)-\varphi_x^-\left(x,0,t\right)\right)=\end{gathered} \tag{8}$$

$$=-\frac{\rho}{\pi}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}(\ddot{w}_k(\tau,t)+V{\dot{w}_k}'(\tau,t))K(x,\tau)d\tau-$$

$$-\frac{V\rho}{\pi}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}(\dot{w}_k(\tau,t)+V{w_k}'(\tau,t))\frac{\partial K(x,\tau)}{\partial x}d\tau,$$

where $x\in\left(a_{2i-1},a_{2i}\right)$, $\tau\neq x$,

$$K(x,\tau)=2ln\left|\frac{\sqrt{(x-c)\,(d-\tau)}+\sqrt{(\tau-c)\,(d-x)}}{\sqrt{(x-c)\,(d-\tau)}-\sqrt{(\tau-c)\,(d-x)}}\right|.$$

The boundary conditions on the ends of plates under $x=a_{2k-1}$ or $x=a_{2k}$ can be:

I. the rigid sealing

$$w_k\left(x,t\right)={w_k}'\left(x,t\right)=u_k\left(x,t\right)=0;$$

II. the articulate sealing

$$w_k\left(x,t\right)={w_k}''\left(x,t\right)=u_k\left(x,t\right)=0;$$

III. the rigid fastening

$$w_k\left(x,t\right)={w_k}'\left(x,t\right)={u_k}'\left(x,t\right)=0;$$

IV. the articulate fastening:

$$w_k\left(x,t\right)={w_k}''\left(x,t\right)={u_k}'\left(x,t\right)+\frac{1}{2}{w_k}'^2\left(x,t\right)=0.$$

Let us introduce the functional

$$\begin{gathered}\Phi\left(t\right)=\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}\left\{M_k(\dot{u}_k^2+\dot{w}_k^2)+E_kJ_k{w_k}''^2+\right. \\ \left.+E_kF_k\left({u_k}'+\frac{1}{2}{w_k}'^2\right)^2\right\}dx+I(t)+J(t),\end{gathered} \tag{9}$$

$$I(t)=\frac{\rho}{\pi}\sum_{i=1}^{n}\sum_{j=1}^{n}\int\limits_{a_{2i-1}}^{a_{2i}}dx\int\limits_{a_{2j-1}}^{a_{2j}}\dot{w}_i(x,t)\dot{w}_j(\tau,t)K(x,\tau)d\tau,$$

$$J(t)=-\frac{\rho V^2}{\pi}\sum_{i=1}^{n}\sum_{j=1}^{n}\int\limits_{a_{2i-1}}^{a_{2i}}dx\int\limits_{a_{2j-1}}^{a_{2j}}{w_i}'(x,t){w_j}'(\tau,t)K(x,\tau)d\tau.$$

Using the inequalities

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\int\limits_{a_{2i-1}}^{a_{2i}}dx\int\limits_{a_{2j-1}}^{a_{2j}}\dot{w}_i(x,t)\dot{w}_j(\tau,t)K(x,\tau)d\tau\geq 0,$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\int\limits_{a_{2i-1}}^{a_{2i}}dx\int\limits_{a_{2j-1}}^{a_{2j}}{w_i}'(x,t){w_j}'(\tau,t)K(x,\tau)d\tau\geq 0,$$

$$\int\limits_{a_{2i-1}}^{a_{2i}}{w_i}''^2(x,t)dx\geq\lambda_{1i}\int\limits_{a_{2i-1}}^{a_{2i}}{w_i}'^2(x,t)dx,\ i=1\div n,$$

$$w_i^2(x,t)\leq(a_{2i}-a_{2i-1})\int\limits_{a_{2i-1}}^{a_{2i}}{w_i}'^2(x,t)dx,\ i=1\div n,$$

where λ_{1i} are least own values of marginal problem [Gahov, 1977; Kollatc, 1968] $\psi^{IV}(x)=-\lambda\psi''(x),\ x\in\left(a_{2i-1},a_{2i}\right),\ i=1\div n$ with boundary conditions, corresponding to mentioned types of fastening, the following theorem is proved on the base of research functional (9)

Theorem 1. Lets assume, that functions $w_k(x,t)$, $u_k(x,t)$ satisfy one of the boundary conditions I - IV and let's execute inequalities

$$\begin{gathered}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}\left(\dot{u}_kg_k\left(t,x,u_k,w_k,\dot{u}_k,\dot{w}_k\right)+\right. \\ \left.+\dot{w}_kh_k\left(t,x,u_k,w_k,\dot{u}_k,\dot{w}_k\right)\right)dx\geq 0,\end{gathered}$$

$$E_kJ_k\lambda_{1k}>\frac{\rho K_kV^2}{\pi},$$

$$K_k=\sup_{x\in(a_{2k-1},a_{2k})}\sum_{i=1}^{n}\int\limits_{a_{2i-1}}^{a_{2i}}K(\tau,x)\,d\tau,\ k=1\div n.$$

Then solution $w_k(x,t)$ equation systems (7), (8) are stability with respect to outraging of the initial values of $\dot{w}_k(x,0)$, $w_k''(x,0)$, $\dot{u}_k(x,0)$, $u_k'(x,0)$ $(k=1\div n)$.

2 Stability of elastic elements of a plane channel

We consider the planar problem about dynamic stability of elastic elements of the walls of an infinitely long channel along which ideal incompressible fluid flows (fig.2).

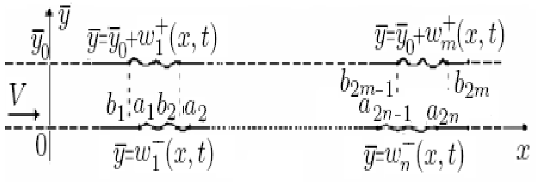

Figure2. Channel.

This problem is formulated in the following way

$$\Delta\varphi = 0,\ (x,y) \in R^2 : |x| < \infty, y \in [0, y_0], \quad (10)$$

$$\begin{gathered}\varphi_y(x, y_0, t) = w_{kt}^+(x,t) + V w_{kx}^+(x,t), \\ x \in (b_{2k-1}, b_{2k}),\ k = 1 \div m,\end{gathered} \quad (11)$$

$$\varphi_y(x, y_0, t) = 0,\ x \in R \backslash \left(\bigcup_{k=1}^{m} [b_{2k-1}, b_{2k}] \right), \quad (12)$$

$$\begin{gathered}\varphi_y(x, 0, t) = w_{kt}^-(x,t) + V w_{kx}^-(x,t), \\ x \in (a_{2k-1}, a_{2k}),\ k = 1 \div n,\end{gathered} \quad (13)$$

$$\varphi_y(x, 0, t) = 0,\ x \in R \backslash \left(\bigcup_{k=1}^{n} [a_{2k-1}, a_{2k}] \right), \quad (14)$$

$$\begin{gathered}\left(\varphi_x^2 + \varphi_y^2\right)_{x=\pm\infty} = 0, (\varphi_t)_{x=-\infty} = 0, \\ y \in (0, y_0),\end{gathered} \quad (15)$$

$$\begin{cases} -E_k^+ F_k^+ \left(u_k^{+\prime} + \frac{1}{2} w_k^{+\prime 2} \right)' + M_k^+ \ddot{u}_k^+ + \\ +g_k^+ \left(t, x, u_k^+, w_k^+, \dot{u}_k^+, \dot{w}_k^+\right) = 0, \\ -E_k^+ F_k^+ \left[w_k^{+\prime} \left(u_k^{+\prime} + \frac{1}{2} w_k^{+\prime 2} \right) \right]' + M_k^+ \ddot{w}_k^+ + \\ +E_k^+ J_k^+ w_k^{+\prime\prime\prime\prime} + h_k^+ \left(t, x, u_k^+, w_k^+, \dot{u}_k^+, \dot{w}_k^+\right) = \\ = -\rho \left(\varphi_t(x, y_0, t) + V \varphi_x(x, y_0, t) \right), \\ x \in (b_{2k-1}, b_{2k}),\quad k = 1 \div m, \end{cases} \quad (16)$$

$$\begin{cases} -E_k^- F_k^- \left(u_k^{-\prime} + \frac{1}{2} w_k^{-\prime 2} \right)' + M_k^- \ddot{u}_k^- + \\ +g_k^- \left(t, x, u_k^-, w_k^-, \dot{u}_k^-, \dot{w}_k^-\right) = 0, \\ -E_k^- F_k^- \left[w_k^{-\prime} \left(u_k^{-\prime} + \frac{1}{2} w_k^{-\prime 2} \right) \right]' + M_k^- \ddot{w}_k^- + \\ +E_k^- J_k^- w_k^{-\prime\prime\prime\prime} + h_k^- \left(t, x, u_k^-, w_k^-, \dot{u}_k^-, \dot{w}_k^-\right) = \\ = \rho \left(\varphi_t(x, 0, t) + V \varphi_x(x, 0, t) \right), \\ x \in (a_{2k-1}, a_{2k}),\quad k = 1 \div n. \end{cases} \quad (17)$$

Here $w_k^{\pm}(x,t)$ and $u_k^{\pm}(x,t)$ are the deflections functions of the plates on the bottom and top walls of the channel respectively.

Using methods of the theory of complex variable functions, the solution of the problem can be reduced to a system of equations for determination of $w_k^{\pm}(x,t)$, $u_k^{\pm}(x,t)$. Using a functional of Liapunov's type, is proved the following theorem

Theorem 2. Lets assume, that functions $w_k^{\pm}(x,t)$, $u_k^{\pm}(x,t)$ satisfy one of the boundary conditions I - IV and let's execute inequalities

$$\begin{gathered} \sum_{k=1}^{m} \int_{b_{2k-1}}^{b_{2k}} \left(\dot{u}_k^+ g_k^+ \left(t, x, u_k^+, w_k^+, \dot{u}_k^+, \dot{w}_k^+\right) + \right. \\ \left. + \dot{w}_k^+ h_k^+ \left(t, x, u_k^+, w_k^+, \dot{u}_k^+, \dot{w}_k^+\right) \right) dx + \\ \sum_{k=1}^{n} \int_{a_{2k-1}}^{a_{2k}} \left(\dot{u}_k^- g_k^- \left(t, x, u_k^-, w_k^-, \dot{u}_k^-, \dot{w}_k^-\right) + \right. \\ \left. + \dot{w}_k^- h_k^- \left(t, x, u_k^-, w_k^-, \dot{u}_k^-, \dot{w}_k^-\right) \right) dx \ge 0, \end{gathered}$$

$$E_k^{\pm} J_k^{\pm} \lambda_{1k}^{\pm} > \frac{\rho V^2 K_{0k}^{\pm}}{\pi},$$

where

$$K_{0k}^+ = \sup_{x \in (b_{2k-1}, b_{2k})} K_1^+(x),\ k = 1 \div m,$$

$$K_{0k}^- = \sup_{x \in (a_{2k-1}, a_{2k})} K_1^-(x),\ k = 1 \div n,$$

$$K_1^{\pm}(x) = \sum_{i=1}^{m} \int_{b_{2i-1}}^{b_{2i}} K^{\pm}(\tau, x)\, d\tau + \sum_{i=1}^{n} \int_{a_{2i-1}}^{a_{2i}} K^{\mp}(\tau, x)\, d\tau,$$

$$K^{\pm}(x, \tau) = ln \left| \frac{e^{\frac{-\pi a_1}{y_0}} + e^{\frac{-\pi b_1}{y_0}}}{e^{\frac{-\pi \tau}{y_0}} \mp e^{\frac{-\pi x}{y_0}}} \right|.$$

Then solution $w_k^+(x,t)$, $k = 1 \div m$, $w_k^-(x,t)$, $k = 1 \div n$, equation systems (10)-(17) are stability with respect to outraging of the initial values of $\dot{w}_k^{\pm}(x,0)$, $w_k^{\pm\prime\prime}(x,0)$, $\dot{u}_k^{\pm}(x,0)$, $u_k^{\pm\prime}(x,0)$.

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