MODELING AND CONTROL OF A UNIROTOR VEHICLE

Omar Sanchez-Rodriguez
CIC
Instituto Politecnico Nacional
Mexico
osanchezrod@gmail.com

Ricardo Barron-Fernandez
CIC
Instituto Politecnico Nacional
Mexico
barron2131@gmail.com

Juan Carlos Chimal-Eguia
CIC
Instituto Politecnico Nacional
Mexico
chimale@cic.ipn.mx

Abstract
This paper addresses the modeling and control of a vehicle unirotor (helicopter). The objective is to model the parameters that influence the system unirotor and raise an control strategy. This paper includes simulations of controlling vertical movement of the vehicle at a desired height.

Key words
UAV's, unirotor, autonomous helicopter, modeling, control of an helicopter, autonomous flight, autonomous takeoff and autonomous landing

1 Introduction

In recent times the UAVs have become a research topic due to the wide variety of applications both in military and civilian areas among others. In the civil area are used in activities such as search and rescue tasks in natural disasters. In the military case they are used in the compilation of information from hostile areas, toxic environments, etc. Another very important application of UAVs are recognition and surveillance missions. A design parameter is the task that play the UAV and of the design and operations depend on the range for different types of missions. The autonomous helicopters since its creation have been used extensively in applications involving aerial photography, cinematography, inspection and other applications. And it is thanks to the recent development of the technologies used on the UAV as the miniaturization of sensors and cameras, as well as of new developments in communication and control systems has been taken great strides in its development. The maneuverability and ability of helicopters and other VTOL configurations for staying stationary in the air are requirements on many of these applications. However, helicopters are more difficult to control than fixed wing aircraft, in fact, require critical stabilization bonds, which are related to displacement behaviors.

During the last decade have been taking efforts by the scientific and technological community oriented stabilization and trajectory tracking of rotary wing aircraft. A lot of universities in the world make efforts in order to develop autonomous vehicles here some examples: the Robotics Institute at Carneige Mellon Univesity conduce an project called autonomous helicopter into which have developed several prototypes of small autonomous aerial vehicles based on the Yamaha R-50 platform, the unirotor called CMU-helicopter won the AUVSI aerial robotics competition.

Figure 1. System model

Southern California University since 1991 leading a project to develop autonomous helicopters, developed several prototypes as AVATAR (Autonomous Vehicle Aerial Tracking and Retrieval / Reconnaissance), which won the AUVSI competition. Berkeley University conduced the Berkeley Aerial Robot project and development the BEAR. Georgia Institute of Technology leads the Unmaned Aerial Vehicle project. In Europe the University of Linkping conduct WITAS project, besides they develop the unirotor based on Yamaha Rmax, the Thecnical University of Berlin had won the Aerial Robotics Competition. All of them have developed many prototypes using different helicopter with a defence law of control and other capabilities.

In this work, we model and propose a control algorithm of an helicopter based on dynamic decoupled. The paper is organized as follows section 1 presents a brief introduction, section 2 presents the mathematical model, section 3 describe an control law, section 4 presents the numerical results and in section 6 we give our conclusions.

2 System Analysis

A complete mathematical model of a helicopter that includes the flexibility of the rotors and the fuselage, the dynamics of the actuators and the engine that drives it, is too complex. In most of the cases, the helicopter is regarded as a rigid body on which the entries are the forces and torques, applied to the center of mass and their outputs are linear position and velocity of the center of mass. From the foregoing relationships involving the fuselage aerodynamic of the vehicle and the effect that they have on some components which act as stabilizers we can neglect that we are working at low speeds. For this case the helicopter and its dinamics shall be considered as a rigid body also if we consider as an ascent-descent vertical stability a desired height, lateral, forward and backward, and we implement a control technique it's possible to improve the stability of the helicopter based on possible errors that arise caused by limitations of sensors. During the hover regime, most unirotores vehicles used a propeller to provide thrust together with an aileron or rudder to control the moving surface in otherwise employing a tail rotor to provide stability. Therefore, the dynamic stability is based entirely on the aerodynamic torques.

2.1 Mathematical Modelling

Let $I = \{i_x^I, j_y^I, k_z^I\}$ inertial reference coordinate and $B = \{i_x^B, j_y^B, k_z^B\}$ the coordinate frame attached to the body of the UAV with origin at its center of gravity, the frame $\Re = \{i_x^{\Re}, j_y^{\Re}, k_z^{\Re}\}$ is considered during movements of tilt and roll.

Let the vector $q = (\xi, \eta)^T \in \mathbb{R}^6$ denotes the generalized coordinates where $\xi = (x, y, z)^T \in \mathbb{R}^3$ translational coordinates with respect to the inertial frame I and $\eta = (\theta, \phi, \psi)^T \in \mathbb{R}^3$ describes the orientation of the vehicle expressed on Euler angles (pitch, roll, yaw) respectively. The transformation matrix that represents the orientation of a UAV of this type is given by $^{B\to I}$

$$R^{B\to I} = \begin{bmatrix} c_\theta c\psi & -c_\theta s\psi & s_\theta \\ c_\phi s_\psi + s_\phi s_\theta c_\psi & c_\phi c_\psi - s_\phi s_\theta s_\psi & -s_\phi c_\theta \\ s_\phi s_\psi - c_\phi s_\theta c_\psi & s_\phi c_\psi + c_\phi s_\theta s_\psi & c_\phi c_\theta \end{bmatrix}$$

Where $s_a = sin(a)$, $c_a = cos(a)$.
There is an auxiliary matrix rotation $R^{\Re\to B}$, this matrix rotation is of the inclination of the rotor blades near the axis $i_x^{\Re}$ is given by,

$$R^{\Re\to I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{bmatrix}$$

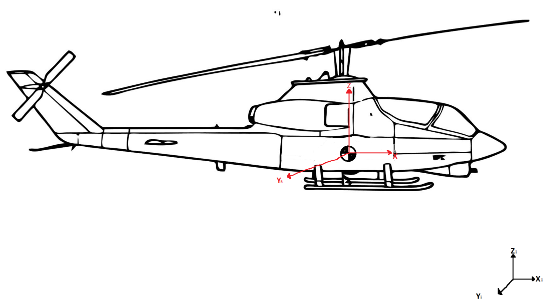

Figure 2. System model

where γ is the angle of the rotor blades and is associated with the roll angle.
The Newton-Euler formulation provides all the equations of motion of a rigid body, which is given by the following vectors expressions

$$\begin{aligned} \bar{m}\dot{v}^B + \Omega \times \bar{m}v^B &= F^B \\ J\dot{\Omega} + \Omega \times J\Omega &= \Gamma^B \end{aligned} \quad (1)$$

where F^B and Γ^B are the force and applied torque respectively to the center of gravity (CG) $\bar{m} = diag(m) \in \mathbb{R}^{3\times 3}$, $m \in \mathbb{R}$ denotes the mass of the vehicle, $\Omega = (p, q, r)^T \in \mathbb{R}^3$ is the angular velocity of the center of mass of the vehicle, $v^B = (u, v, w)^T \in \mathbb{R}^3$ is the translational speed of the center of mass of the vehicle and $J \in \mathbb{R}^{3\times 3}$ contains the moments of inertia of the vehicle.

2.1.1 Translational movement

The translatory movement of the UAV relative to the body is described by the following vector equation:

$$T^B = R^{\Re\to B}T^\gamma \quad (2)$$

$$\bar{m}\dot{v}^B + \Omega \times \bar{m}v^B = R^{B\to I}mG^I + T^B \quad (3)$$

where $G^I = (0, 0, -g) \in \mathbb{R}^3$ is the gravity vector and $T^\gamma = (0, 0, T_r)^T \in \mathbb{R}^3$ is the vector of rotor thrust. The translational dynamics relative to inertial frame is obtained from the following expression:

$$\dot{\xi} = v \quad (4)$$

$$m\dot{v}^I = mG^I + R^{B\to I}T^B \quad (5)$$

then

$$
\begin{aligned}
\ddot{x} &= T_r c_\gamma s_\theta - T_r s_\gamma c_\theta s_\psi \\
\ddot{y} &= T_r s_\gamma c_\phi c_\psi - T_r s_\gamma s_\phi s_\theta s_\psi - T_r s_\gamma c_\phi s_\theta \\
\ddot{z} &= T_r s_\gamma s_\phi c_\psi + T_r s_\gamma c_\phi s_\theta s_\psi + T_r s_\gamma c_\phi c_\theta - mg
\end{aligned} \tag{6}
$$

2.1.2 Rotation Movement Corresponds to the torques applied on the rigid body, these torques are:

1. **Actuator torque** this is the torque provided by the actuators and are described by the following vector expression:

$$\overrightarrow{\Gamma}_c = \overrightarrow{l} \times F \tag{7}$$

then

$$\Gamma_c = \begin{bmatrix} -l_r T_r s_\gamma \\ l_p(T_r) \\ l_a(T_r) \end{bmatrix}$$

2. **Gyroscopic torque** this is caused by the inclination of the rotor blade is given by the following vector expression:

$$\overrightarrow{\Gamma}_g = -I_r(\Omega \times \omega_r) \tag{8}$$

then

$$\Gamma_g = \begin{bmatrix} r\omega_r s_\gamma - q\omega_r c_\gamma \\ p\omega_r c_\gamma \\ -p\omega_r s_\gamma \end{bmatrix}$$

Where I_r is the moment of inertia of the propeller and ω_r denotes the angular velocity of the rotor.

3. **Torque weight** is the torque provided by the pendular mass is described by the following vector expression:

$$\overrightarrow{\Gamma}_w = -l_r \times R^{B \to I} m G^I \tag{9}$$

then

$$\Gamma_w = \begin{bmatrix} -mgl_r(c_\phi s_\psi + c_\phi s_\theta s_\psi) \\ -mgl_r(c_\phi s_\theta c_\psi - s_\theta s_\psi) \\ 0 \end{bmatrix}$$

Then, the total external torque in the body frame is described by the following expression:

$$\Gamma^B = \Gamma_c + \Gamma_g + \Gamma_w = \begin{bmatrix} \tau_M \\ \tau_L \\ \tau_N \end{bmatrix}$$

Thus, the rotational dynamics in terms of the generalized coordinates is given by:

$$\ddot{\eta} = (JW_n)^{-1}(-J\dot{W}_n\dot{\eta} - \Omega \times J\Omega + \Gamma^B) \tag{10}$$

where $W_n \in \mathbb{R}^3$ is an orthonormal transformation and Ω is the result of the projection of the vector $\dot{\eta}$ generated per rotation. Equation 10 can be written as,

$$
\begin{aligned}
\ddot{\phi} &= \frac{1}{c_\theta c_\psi}(s_\theta c_\psi \dot{\phi}\dot{\theta} + c_\theta s_\psi \dot{\phi}\dot{\psi} - c_\psi \dot{\theta}\dot{\psi} - s_\psi \ddot{\theta}) \\
&\quad + \frac{1}{I_x c_\theta c_\psi}[-qr(I_z - I_y) + \tau_M] && (11) \\
\ddot{\theta} &= \frac{1}{c_\psi}(-s_\theta s_\psi \dot{\phi}\dot{\theta} + c_\theta c_\psi \dot{\phi}\dot{\psi} + s_\psi \dot{\theta}\dot{\psi} + c_\theta s_\psi \ddot{\phi}) \\
&\quad + \frac{1}{I_y c_\psi}[-pr(I_x - I_z) + \tau_L] && (12) \\
\ddot{\psi} &= -c_\theta \dot{\phi}\dot{\theta} - s_\theta \ddot{\phi} + \frac{1}{I_z}[-pq(I_y - I_x)] \\
&\quad + \tau_N && (13)
\end{aligned}
$$

with,

$$
\begin{aligned}
\tau_M &= u_\phi + (qw_r c_\gamma - rw_r s_\gamma) \\
&\quad - mgl_r(c_\psi s_\phi + c_\phi s_\theta s_\psi) && (14) \\
\tau_L &= u_\theta - pw_r c_\gamma - mgl_r(c_\phi s_\theta c_\psi - s_\phi s_\psi) && (15) \\
\tau_M &= u_\psi + pw_r s_\gamma && (16)
\end{aligned}
$$

where,

$$
\begin{aligned}
u_\phi &= -l_r T_r s_\gamma && (17) \\
u_\theta &= l_p T_r && (18) \\
u_\psi &= l_a T_r && (19)
\end{aligned}
$$

For control analysis purposes, the full nonlinear model described by the equations 4, 5 and 10 of 6 DOF can be reduced if we take into account the following assumptions and physical facts:

1. The magnitude of the force representing the weight of the UAV is smaller than the lift forces and thrust.
2. Only the aerodynamic force is generated by the deflection of the control surfaces.
3. The gyroscopic torque generated by the inclination of a propeller during the roll control is disregarded.
4. The wing lift force is disregarded, due to the combination of the symmetry of the wing and the direction of air flow generated by the propeller, which coincides with the line of wing stall.
5. The inertial tensor matrix J and the vehicle mass m are standardized.

Considering the above a nonlinear system 6 DOF described by equationS 4, 5 and 10 is divided into three sets of equations: subsystems lateral, longitudinal and axial.
Lateral Subsystem is obtained the set of equations with $\theta = \psi = 0$ it is to regulate the angle of roll ϕ and is given by:

$$\begin{aligned} \ddot{y} &= -T_r sin(\phi - \gamma) \\ \ddot{z} &= T_r cos(\phi - \gamma) - mg \\ \ddot{\phi} &= u_\phi - mgl_r sin(\phi) \end{aligned} \tag{20}$$

Longitudinal Subsystem: The longitudinal subsystem is the result of pitch angle control θ with $\phi = \psi = \gamma = 0$ and is given by:

$$\begin{aligned} \ddot{x} &= T_r sin(\theta) \\ \ddot{z} &= T_r cos(\theta) - mg \\ \ddot{\theta} &= u_\theta - mgl_r sin(\theta) \end{aligned} \tag{21}$$

Axial Subsystem: This subsystem is the result to obtain the controlling of the yaw angle ψ with $\phi = \theta = \gamma = 0$ i.e.,

$$\ddot{\psi} = u_\psi \tag{22}$$

3 Controller design

Now we describe the control algorithm to stabilize unirotor hovering. To this purpose we develop a control algorithm for each of the decoupled dynamic systems in longitudinal, lateral and axial parts. Parts generating a six degree of freedom nonlinear system.
Let us analyze the longitudinal dynamics equation 21, the control input that stabilizes the unirotor vertical position is described by the equation

$$T_r = \frac{(r + mg)}{cos(\theta)} \tag{23}$$

where

$$r = -k_{z1}\dot{z} - k_{z2}(z - z_d) \tag{24}$$

$m = 1$, z_d is a desired height and k_{z1} and k_{z2} are positive constants. Now substituting equation 23 and equation 24 in equation 21and considering that $z \to z_d$, $\dot{z} \to 0$ and $r \to \infty$ when $t \to 0$ we obtain the following reduced system,

$$\begin{aligned} \ddot{x} &= gtan(\theta) \\ \ddot{\theta} &= u_\theta - gl_r sin(\theta) \end{aligned} \tag{25}$$

assuming that $tan(\theta) \equiv \theta$ and $\sin(\theta) \equiv \theta$, the equations 25 reduce to,

$$\begin{aligned} \ddot{x} &= g\theta \\ \ddot{\theta} &= u_\theta - gl_r\theta \end{aligned} \tag{26}$$

the above equation can be rewritten as

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= g\theta_1 \\ \dot{\theta_1} &= \theta_2 \\ \dot{\theta_2} &= u_\theta g l_r \theta_1 \end{aligned} \tag{27}$$

Now we will deal with the problem of stabilize the lateral dynamics equation 21. Assuming $T_r \equiv g$, $sin(\phi) \equiv \phi$ and $sin(\gamma) \equiv \gamma$ since ϕ and γ are relatively small, and we obtain

$$\begin{aligned} \ddot{y} &= -g\phi \\ \ddot{\phi} &= u_\phi - gl_r sin(\phi) \end{aligned} \tag{28}$$

Analyzing also the axial system equation 22, we propose the following control algorithm

$$u_\psi = -k_{\psi1}\dot{\psi} - k_{\psi2}\psi \tag{29}$$

where $k_{\psi1}$and $k_{\psi2}$ are positive constants. Now substituting equation 29 in equation22 we obtain,

$$\ddot{\psi} = -k_{\psi1}\dot{\psi} - k_{\psi2}\psi \tag{30}$$

4 Simulations Results

We show the results of the numerical simulation model in 6 DOF vehicle in hover and unsteady, showing a linear controller performance to stabilize the position and orientation of the UAV. The parameters used for the simulations are: $k_{z1} = 1$ and $k_{z2} = 1.8$; $k_{\psi1} = 4$ and $k_{\psi2} = 4$; $k_1 = 2$, $k_2 = 4$, $k_3 = 1$ and $k_4 = 3$. Experiments were performed with a numerical controller designed. The experiment was performed with the closed loop system and the evolution of its position depicted in figure 3, it shows that the vehicle is in a vicinity of stability during the ascent stage, for the same experiment the figure 4 shows the orientation also remains stable. This experiment was done using the following initial conditions $x(0) = 0.1$, $y(0) = 0.1$, $z(0) = 0.1$, $\phi(0) = \pi/10$,$\theta(0) = \pi/10$, $\psi(0) = \pi/10$ and a desired height of $1m$. The figure 5 shows the evolution of its control signal.

5 Conclusion

In this paper we presents an option for control the unirotor system. The advantages are model is simplified and in concecuence the control law is more simply. Considering the above the principal impact is in order best control law performance.

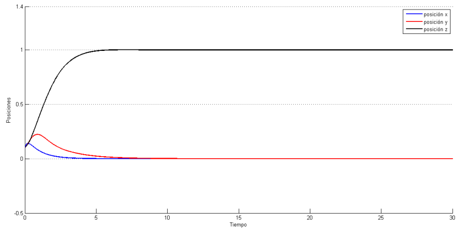

Figure 3. Positions

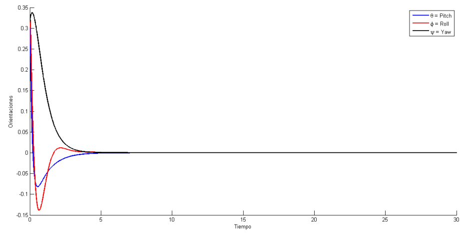

Figure 4. Euler's orientations

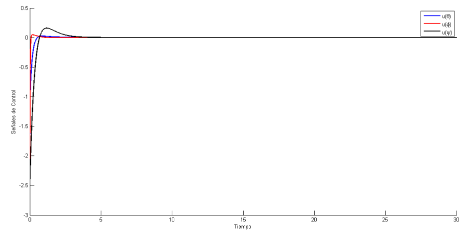

Figure 5. Signals control

6 Acknowledgements

This work was done with support from CONACYT Mexico, COFAA IPN, EDI IPN, also work is associated with the projects SIP-20131599 and SIP-20131210. And is also associated with project SENER HIDROCARBUROS 146515.

References

Altug, E., Ostrowski, J.P. and Taylor, C.J. (2005) Control of a quadrotor helicopter using dual camera visual feedback. *The International Journal of Robotics Research*, **25/5**, pp. 329–341.

Bishop, R.H., Dorf, R.C. (2005).*Sistemas de Control Moderno*. Pearson. USA.

Chi-Tsong, C. (1970). *Introduction to linear systems*. Rinehart and Winston line. New York USA

Chi-Tsong, C. (1999). *Linear System Theory and Design*. Oxford Uninversity Press. New York, USA.

Chowdhary, G. and Lorenz, S. (2005). Control of a VTOL UAV via Online Parameter Estimation. In *Guidance, Navigation, and Control Conference and Exhibit.*

Close, C. and Frederick, D. *Modeling and Analysis of Dynamic Systems*. Jonh Wiley Sons Inc. USA.

Dominguez, S., et all. *Control en el espacio de estados*. Prentice Hall. Madrid, Espaa.

Heredia, G., Ollero, A., Bejar, M. and Mahtani, R. (2008). Sensor and actuator fault detection in small autonomous helicopters. *Journal of IFAC of Mechatronics The Science of Intelligent Machines.*, **18**, pp. 90–99.

Hinrichsen, D. and Pritchard, A. *Mathematical Systems Theory Modeling State Space Analysis Stality and Robustness*. Springer. Germany.

Khalil, H. *Non linear systems*. Prentice Hall. USA.

Lozano, R. and et all. Real-time stabilization and tracking of a four mini rotorcraft. *IEEE Transactions on Control Systems Technology.*,**12**, pp. 510–516.

Pounds, and et al.(2006) Nonlinear control for systems with bounded inputs: Real-time embedded control applied to UAVs. In *IEEE Proceedings of the 45th Conference on Decision and Control* San Diego California, E.U.A, 2006. pp. 5888–5893.

Vissire, D. and Petit, N. (2008) An embedded system for small-scaled autonomous vehicles. In *International Conference on Informatics in Control, Automation and Robotics.* pp. 158–164.